Lecture 5

2022/2023

Microwave Devices and Circuits for Radiocommunications

2022/2023

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 12-14, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 21-28.02.2023 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2022/2023

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 21.02.2022)
    - personal homework

Materials

- http://rf-opto.etti.tuiasi.ro

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, “Microwave Engineering”**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by **email**/online exam
  - used at lectures/laboratory

Photos

Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Trimite email acestui student | Adauga acest student la lista (0)

Detalii curente

Finantare	Buget
Bursa	Fara Bursa

Observatii

Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5244

Trimite email acestui student | Adauga acest student la lista (0)

Detalii curente

Finantare	Buget
Bursa	Bursa de Studii

Observatii

Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Acceseaza ca acest student

Note obtinute

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

Profile photo

- Profile photo – online “exam”

Examene online: 2020/2021

Disciplina: MDC (Microwave Devices and Circuits (Engleza))

Pas 3

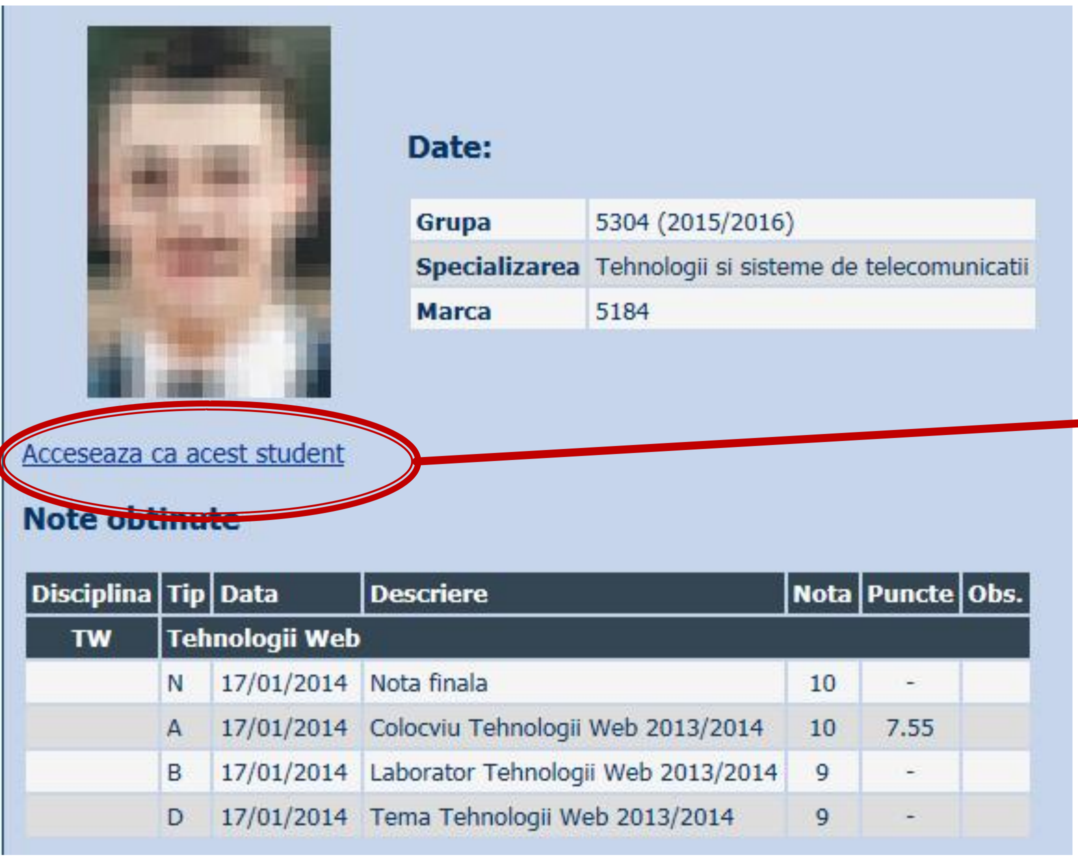
Nr.	Titlu	Start	Stop	Text
1	Profile photos	03/03/2021; 10:00	08/04/2021; 08:00	Online "exam" created f ..
2	Mini Test 1 (lecture 2)	03/03/2021; 15:35	03/03/2021; 15:50	The current test consis ..

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Acceseaza ca acest student

Access

- ~~Not customized~~



Date:

Grupa	5304 (2015/2016)
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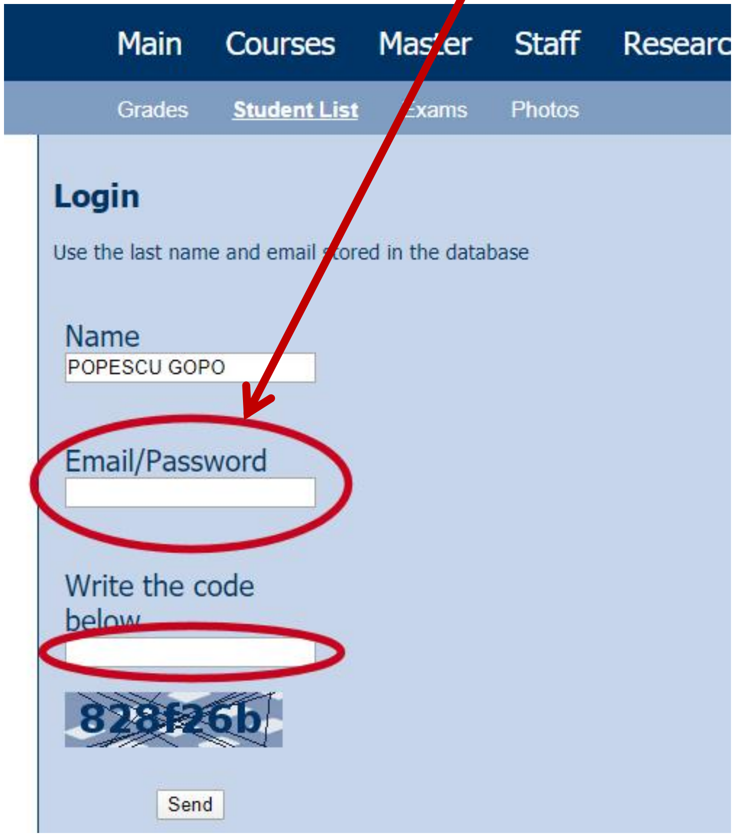
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

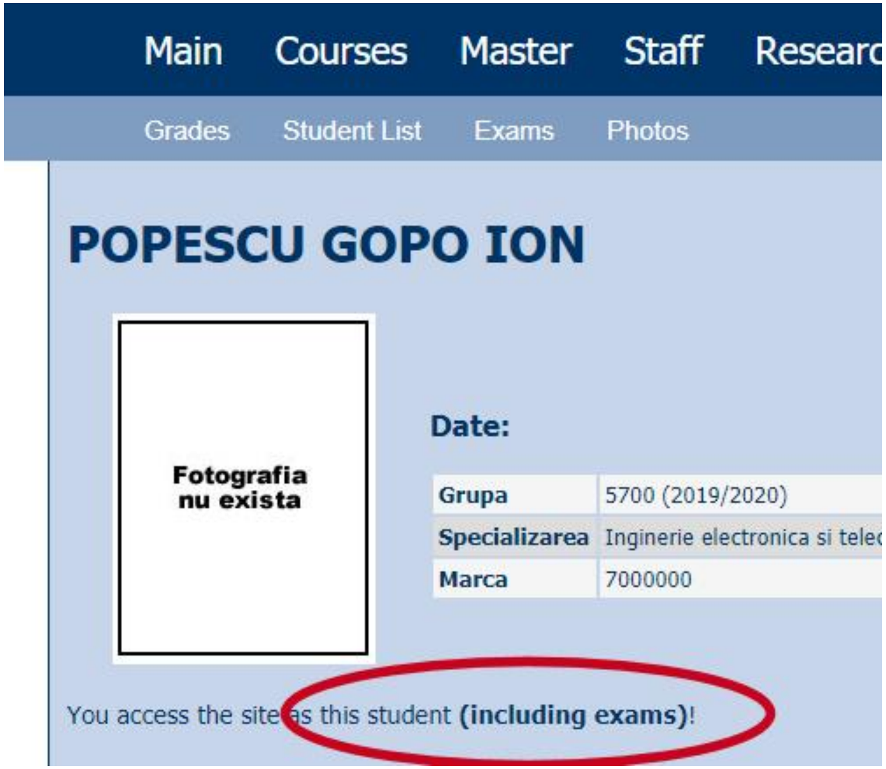
Online

- access to **online exams** requires the **password** received by email



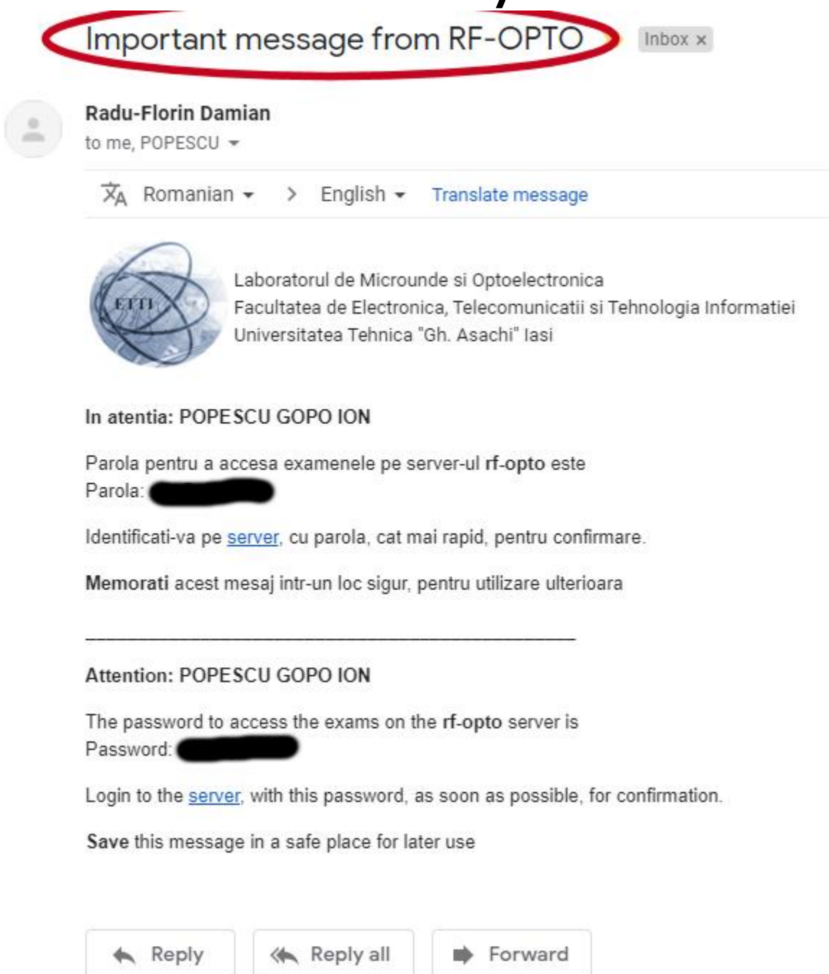
Online

- access email/password



Password

- received by email


Important message from RF-OPTO Inbox ×

Radu-Florin Damian
to me, POPESCU

Romanian > English Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

---

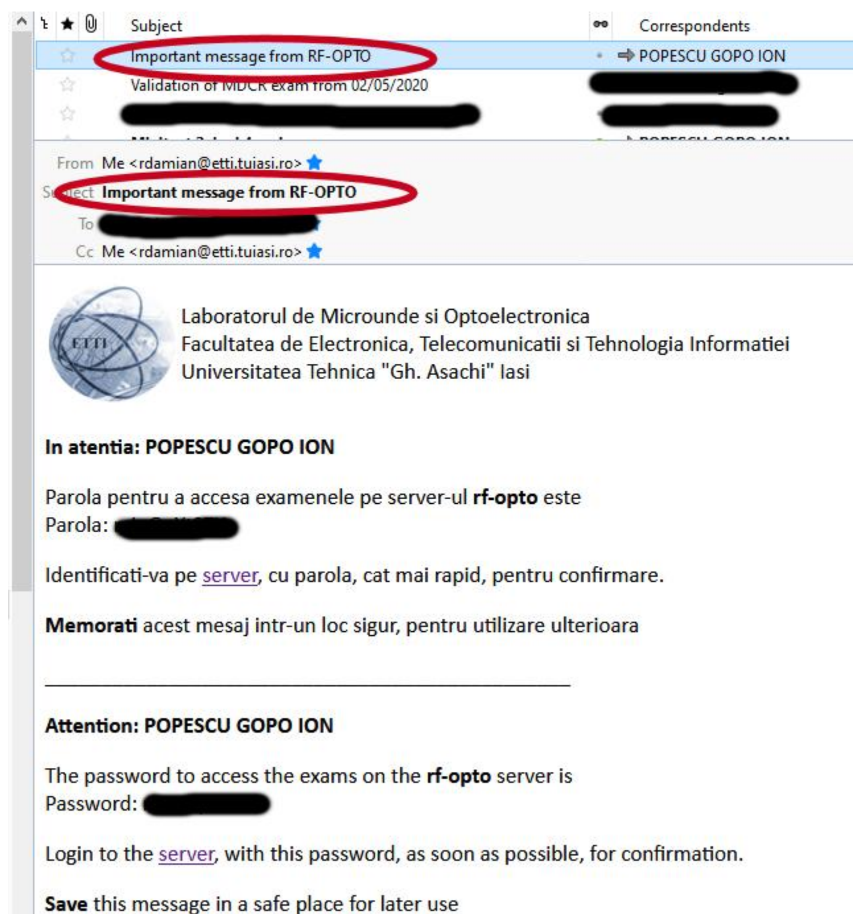
Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply | Reply all | Forward


Subject	Correspondents
Important message from RF-OPTO	POPESCU GOPO ION
Validation of MDCR exam from 02/05/2020	[redacted]
[redacted]	[redacted]

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

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Password: [redacted]

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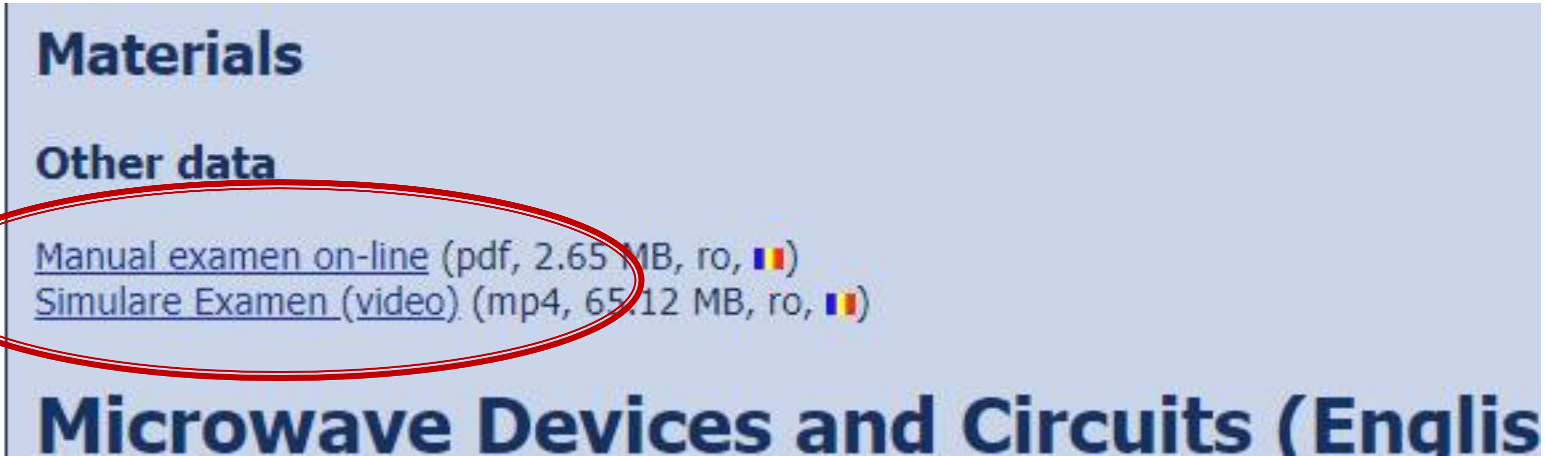
Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~


Materials

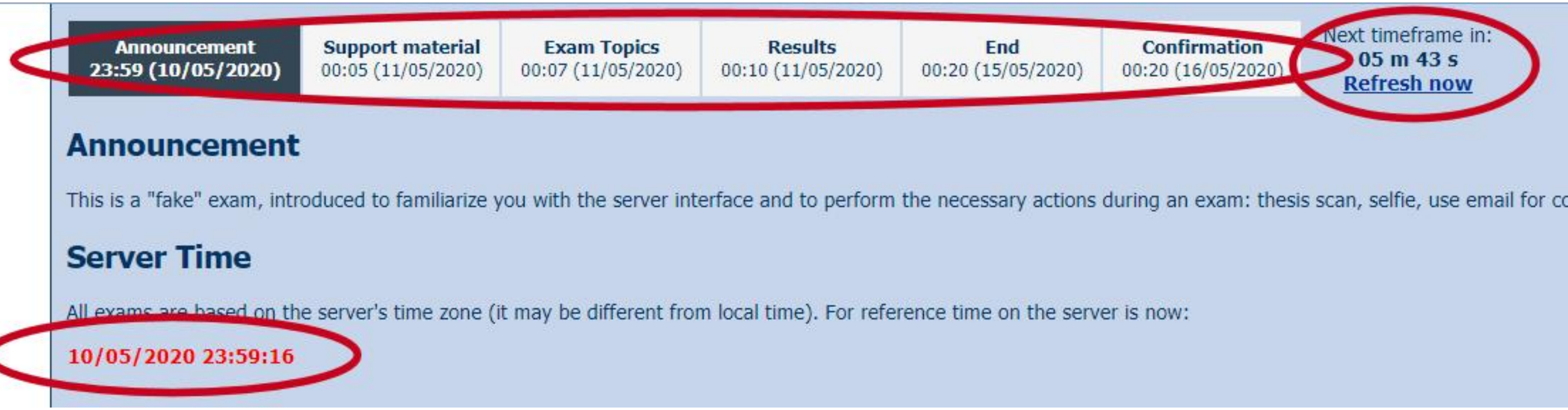
Other data

Manual examen on-line (pdf, 2.65 MB, ro,)
Simulare Examen (video) (mp4, 65.12 MB, ro,)

Microwave Devices and Circuits (Englis

Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri s	T2, fisier parmetri s	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

Online results submission

Grade = Quality of the work +

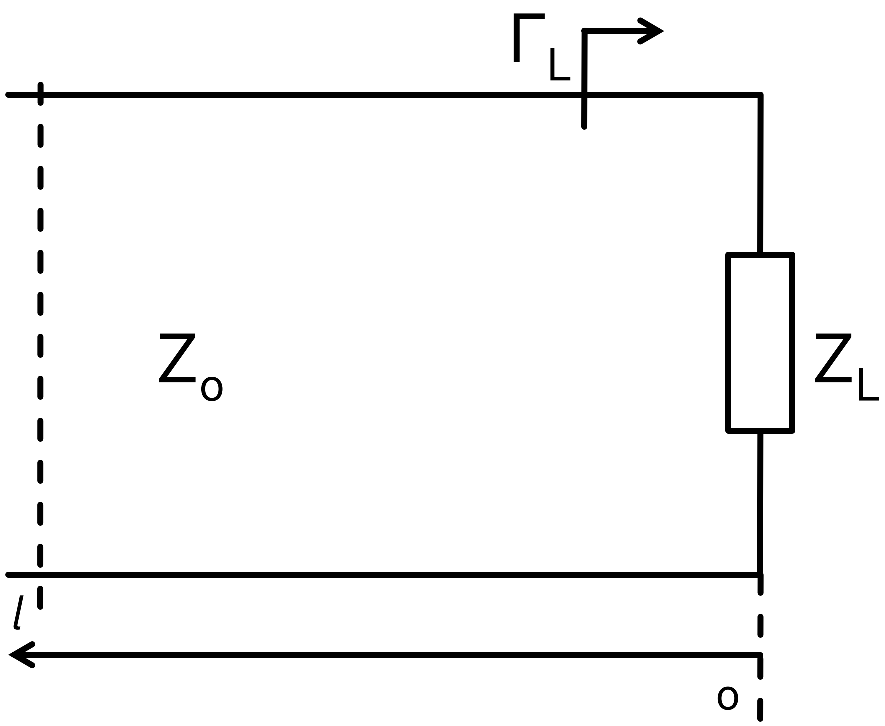
+ Quality of the submission

TEM transmission lines

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

The lossless line



$$V(z)=V_0^{+}e^{-j\cdot\beta\cdot z}+V_0^{-}e^{j\cdot\beta\cdot z}$$

$$I(z)=\frac{V_0^{+}}{Z_0}e^{-j\cdot\beta\cdot z}-\frac{V_0^{-}}{Z_0}e^{j\cdot\beta\cdot z}$$

$$Z_L=\frac{V(0)}{I(0)} \qquad Z_L=\frac{V_0^{+}+V_0^{-}}{V_0^{+}-V_0^{-}}\cdot Z_0$$

- voltage reflection coefficient

$$\boxed{\Gamma=\frac{V_0^{-}}{V_0^{+}}=\frac{Z_L-Z_0}{Z_L+Z_0}}$$

- Z_o real

The lossless line

$$V(z)=V_0^+\cdot\left(e^{-j\cdot\beta\cdot z}+\Gamma\cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z)=\frac{V_0^+}{Z_0}\cdot\left(e^{-j\cdot\beta\cdot z}-\Gamma\cdot e^{j\cdot\beta\cdot z}\right)$$

- time-average Power flow along the line

$$P_{avg}=\frac{1}{2}\cdot\mathrm{Re}\left\{V(z)\cdot I(z)^*\right\}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\mathrm{Re}\left\{1-\Gamma^*\cdot e^{-2j\cdot\beta\cdot z}+\Gamma\cdot e^{2j\cdot\beta\cdot z}-\left|\Gamma\right|^2\right\}$$

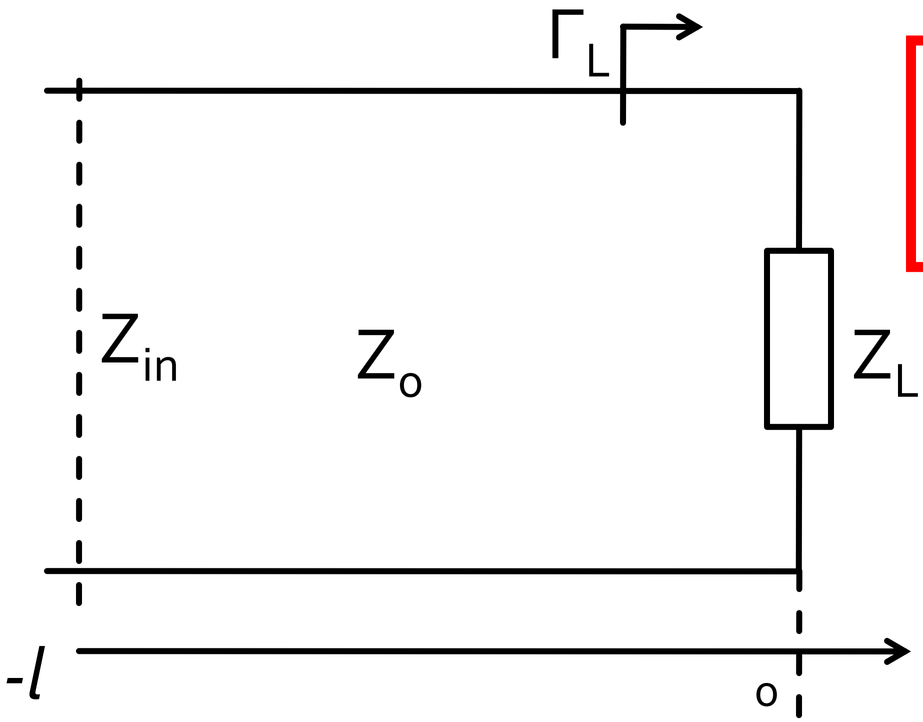
$$\left(z-z^*\right)=\mathrm{Im}$$

$$P_{avg}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\left(1-\left|\Gamma\right|^2\right)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] $\quad \mathrm{RL}=-20\cdot\log\left|\Gamma\right| \quad [\mathrm{dB}]$

The lossless line

- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

General theory

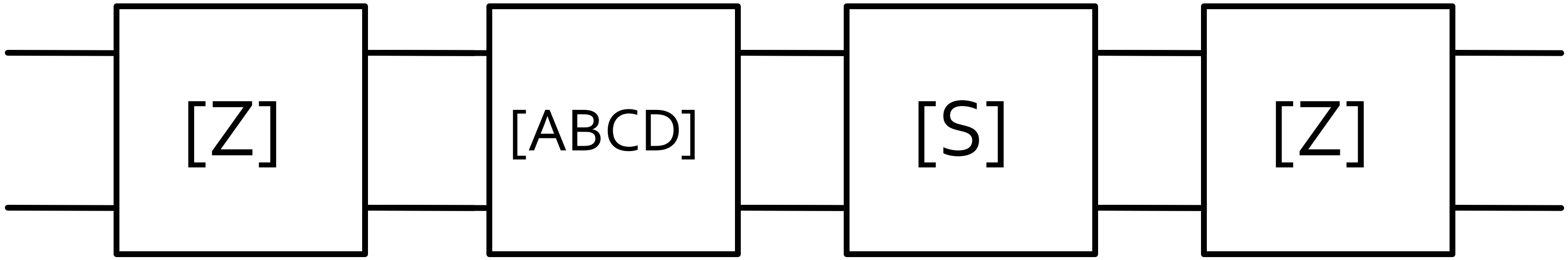
Microwave Network Analysis

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Network Analysis

- We try to separate a complex circuit into individual blocks
- These are analyzed separately (decoupled from the rest of the circuit) and are characterized only by the port level signals (**black box**)
- Network-level analysis allows you to put together individual block results and get a total result for the entire circuit



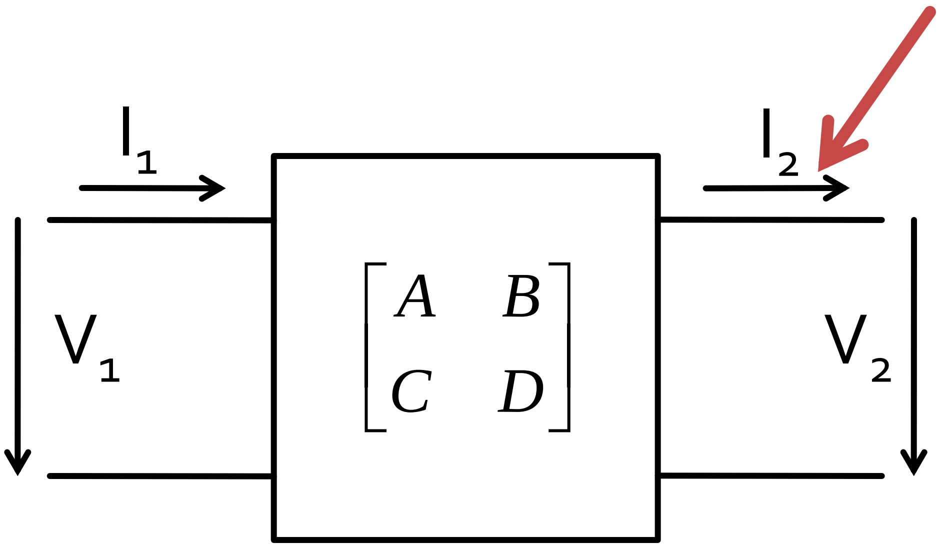
Network Analysis

- Each matrix is best suited for a particular mode of port excitation (V, I)
  - matrix H in common emitter connection for TB: I_B, V_{CE}
  - matrices provide the associated quantities depending on the "attack" ones
- Traditional notation of Z, Y, G, H parameters is in lowercase (z, y, g, h)
- In microwave analysis we prefer the notation in uppercase to avoid confusion with the **normalized parameters**

$$z = \frac{Z}{Z_0} \qquad y = \frac{Y}{Y_0} = \frac{1/Z}{1/Z_0} = \frac{Z_0}{Z} = Z_0 \cdot Y$$

$$z_{11} = \frac{Z_{11}}{Z_0} \qquad y_{11} = \frac{Y_{11}}{Y_0} = Z_0 \cdot Y_{11}$$

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

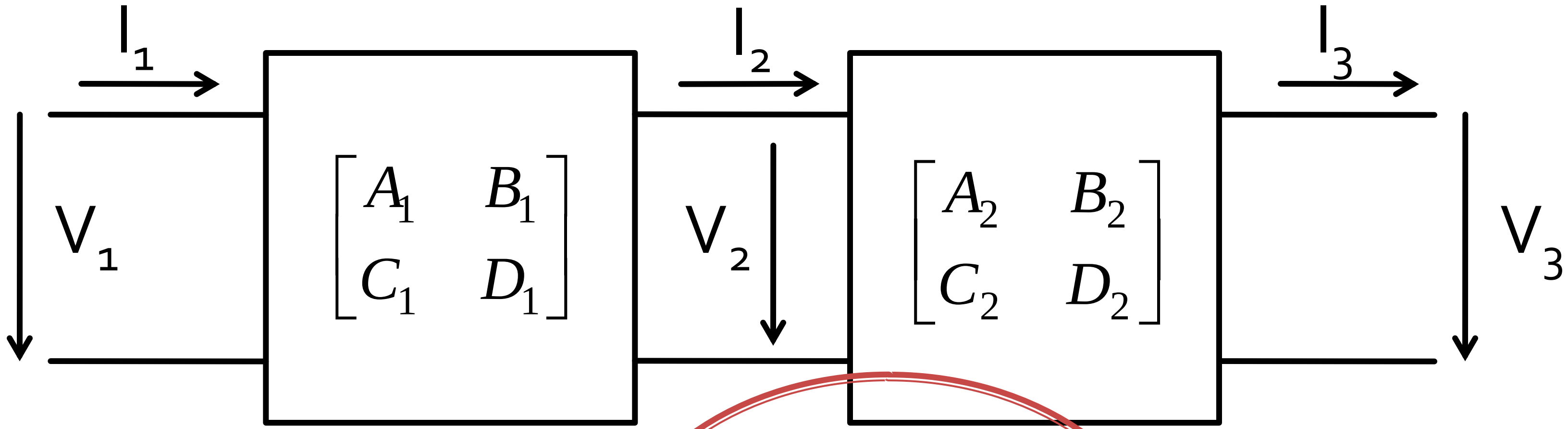
$$V_1 = A \cdot V_2 + B \cdot I_2$$
$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2 = 0} \qquad B = \left. \frac{V_1}{I_2} \right|_{V_2 = 0} \qquad C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0} \qquad D = \left. \frac{I_1}{I_2} \right|_{V_2 = 0}$$

ABCD (transmission) matrix

- This 2X2 matrix characterizes the "input"/"output" relation
- Allows easy chaining of multiple two-ports



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Library of ABCD matrices

TABLE 4.1 ***ABCD* Parameters of Some Useful Two-Port Circuits**

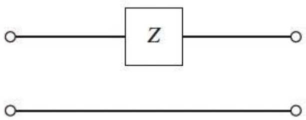
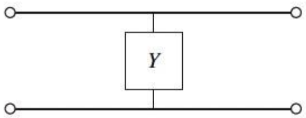
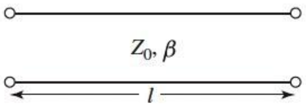
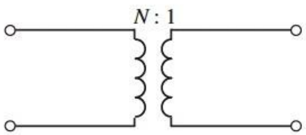
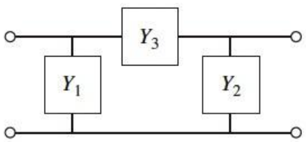
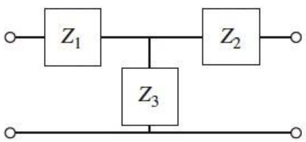
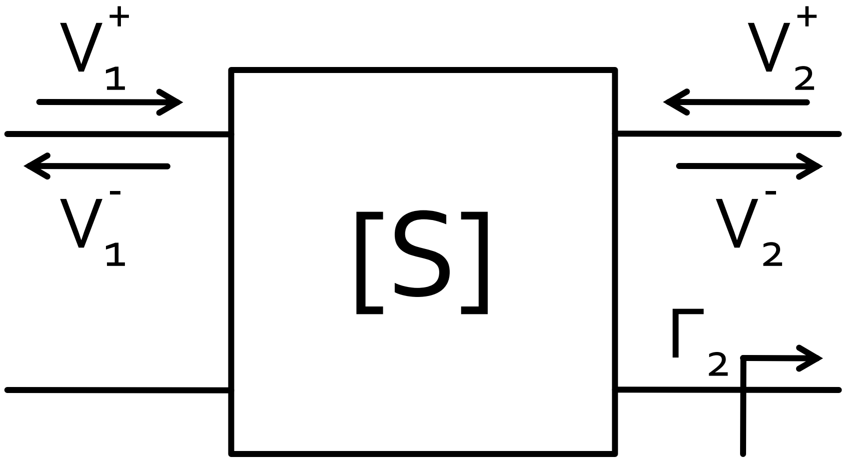
Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta\ell$ $C = jY_0 \sin \beta\ell$	$B = jZ_0 \sin \beta\ell$ $D = \cos \beta\ell$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

Table 4.1

Scattering matrix – S

- Scattering parameters



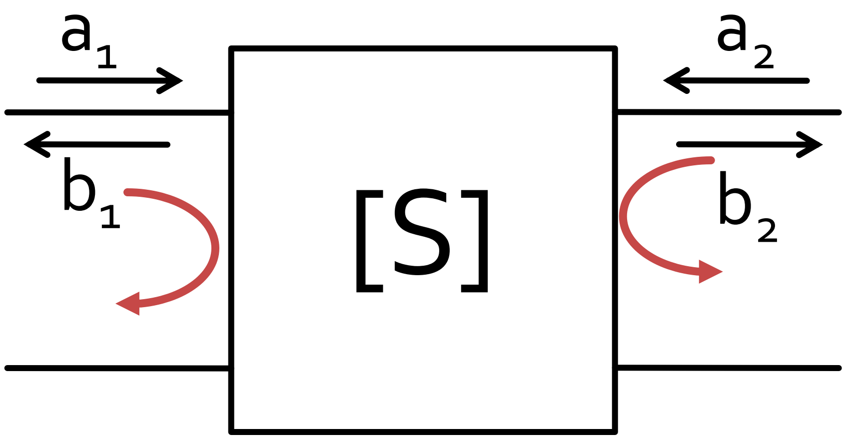
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S

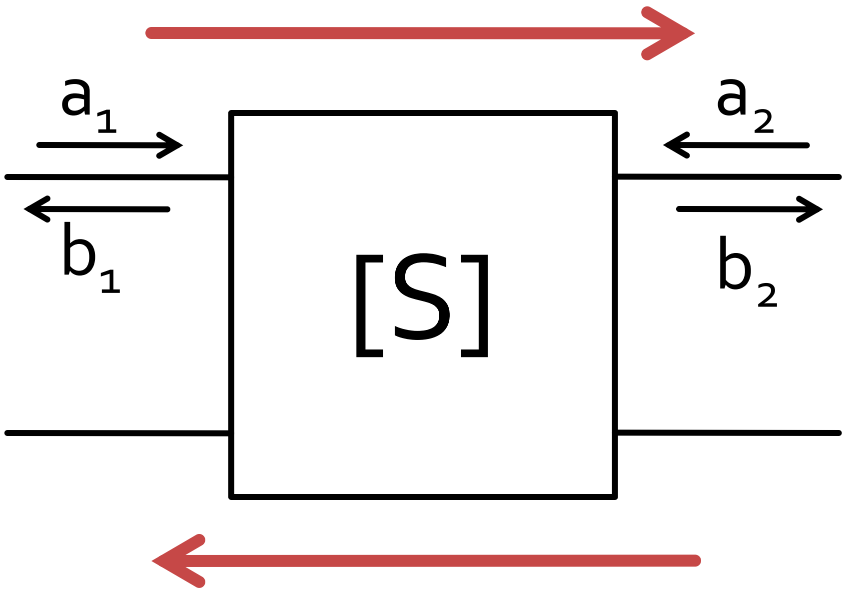


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

- S_{11} and S_{22} are reflection coefficients at ports 1 and 2 when the other port is matched

Scattering matrix – S

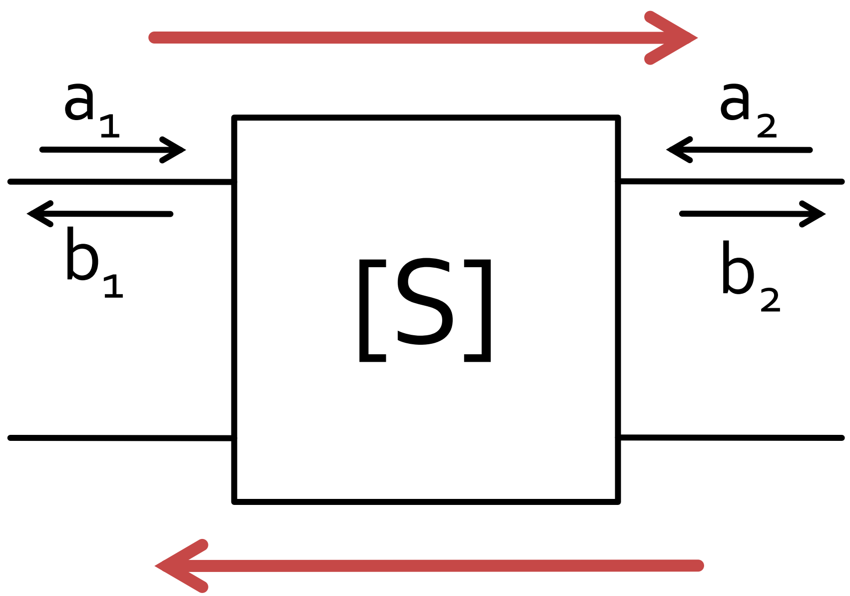


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude gain when the other port is matched

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Measuring S parameters - VNA

- Vector Network Analyzer

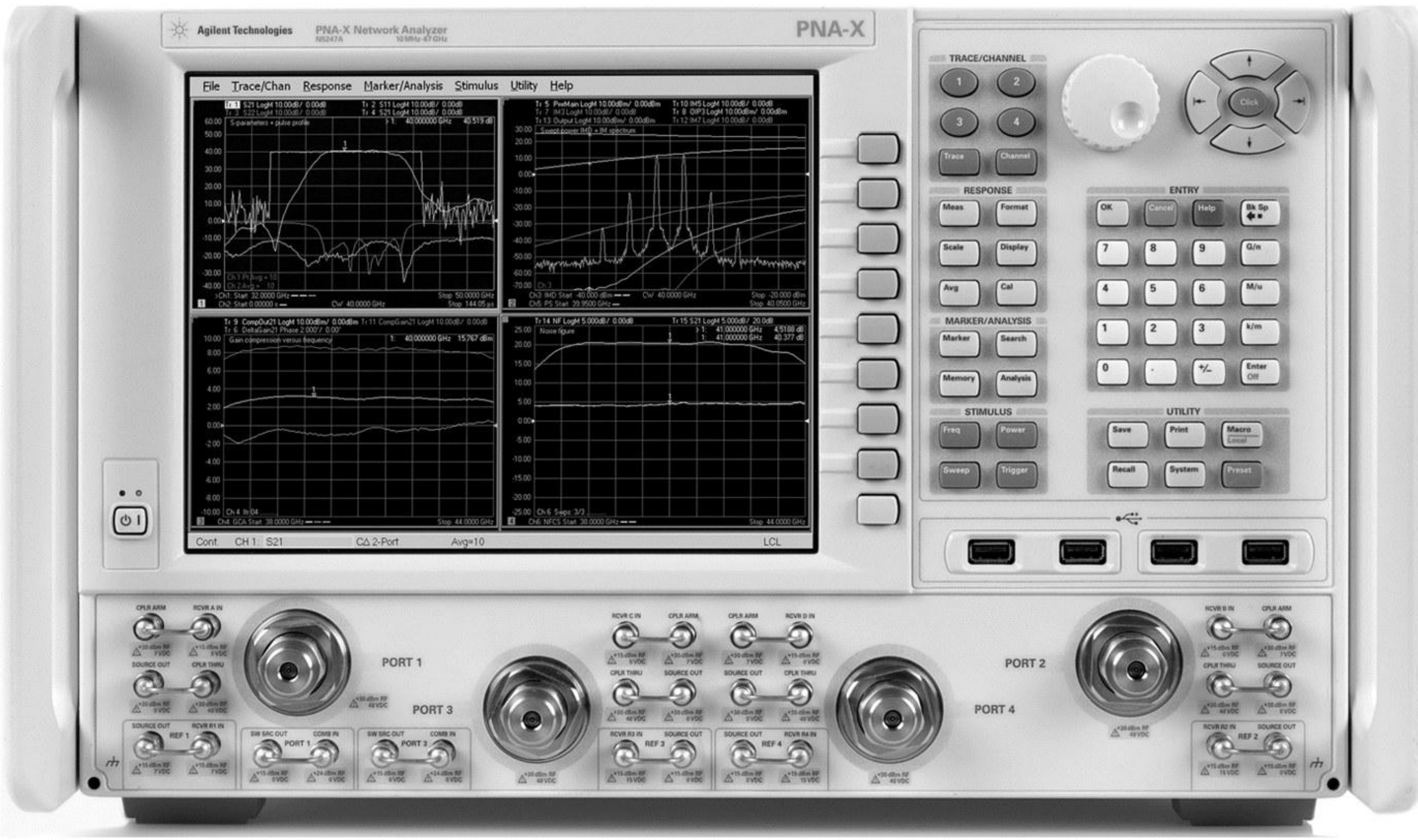
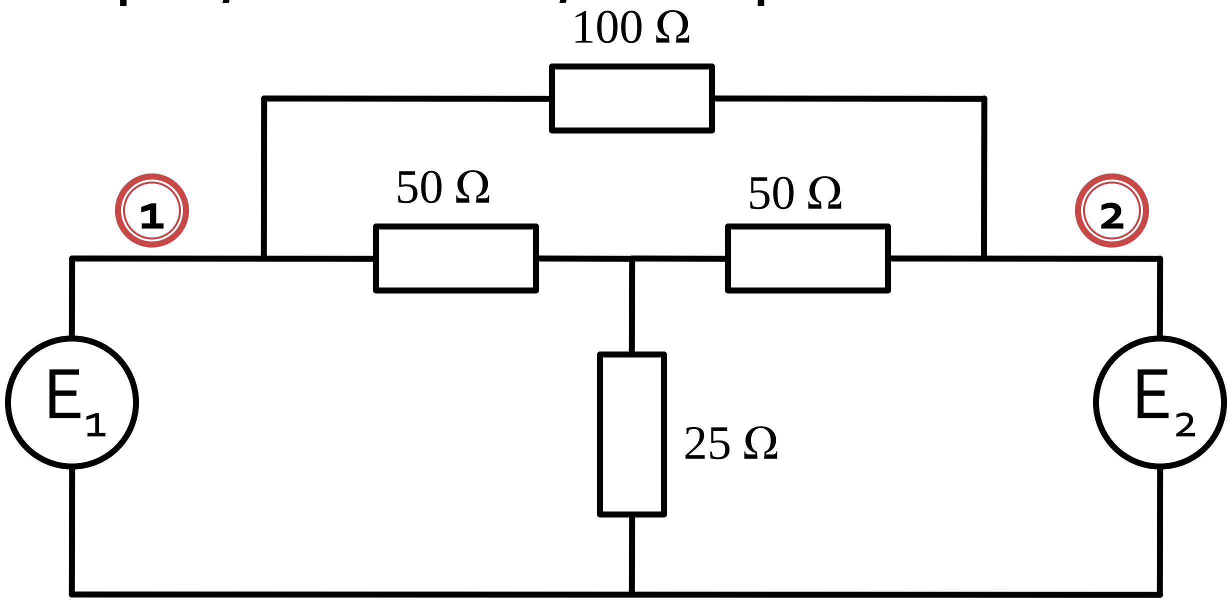


Figure 4.7
Courtesy of Agilent Technologies

Even/Odd Mode Analysis

Even/Odd Mode Analysis

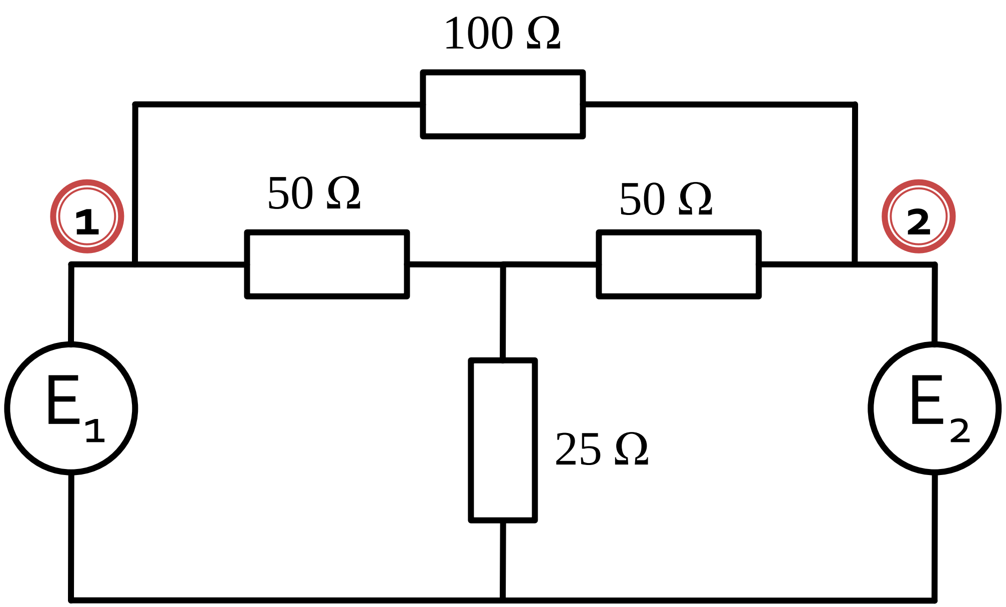
- useful method, necessary even for multiple ports
- example, resistors, two port circuit

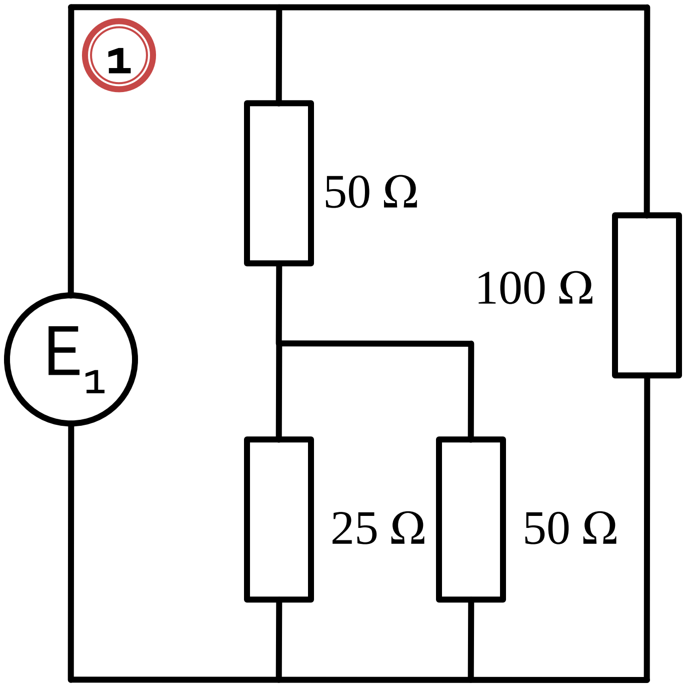


Even/Odd Mode Analysis

- assume we want to compute Y_{11}
- $E_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$



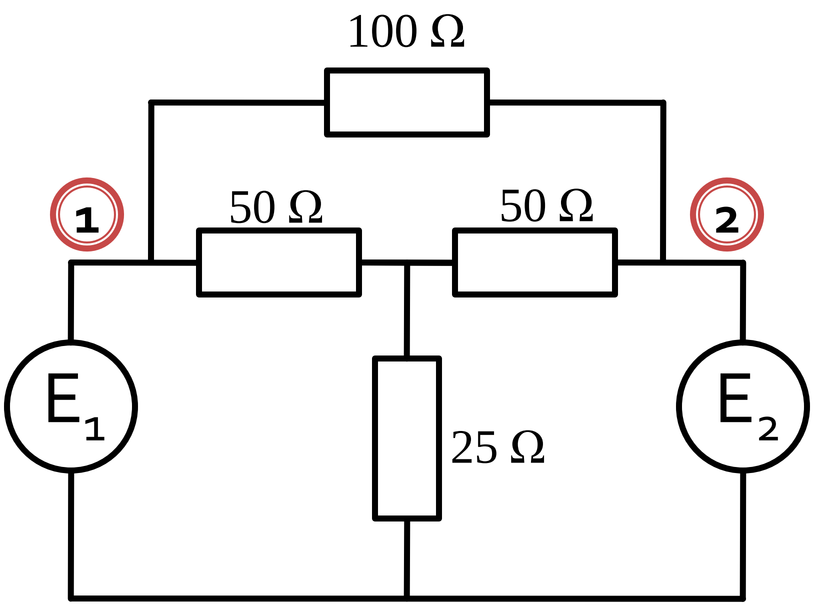


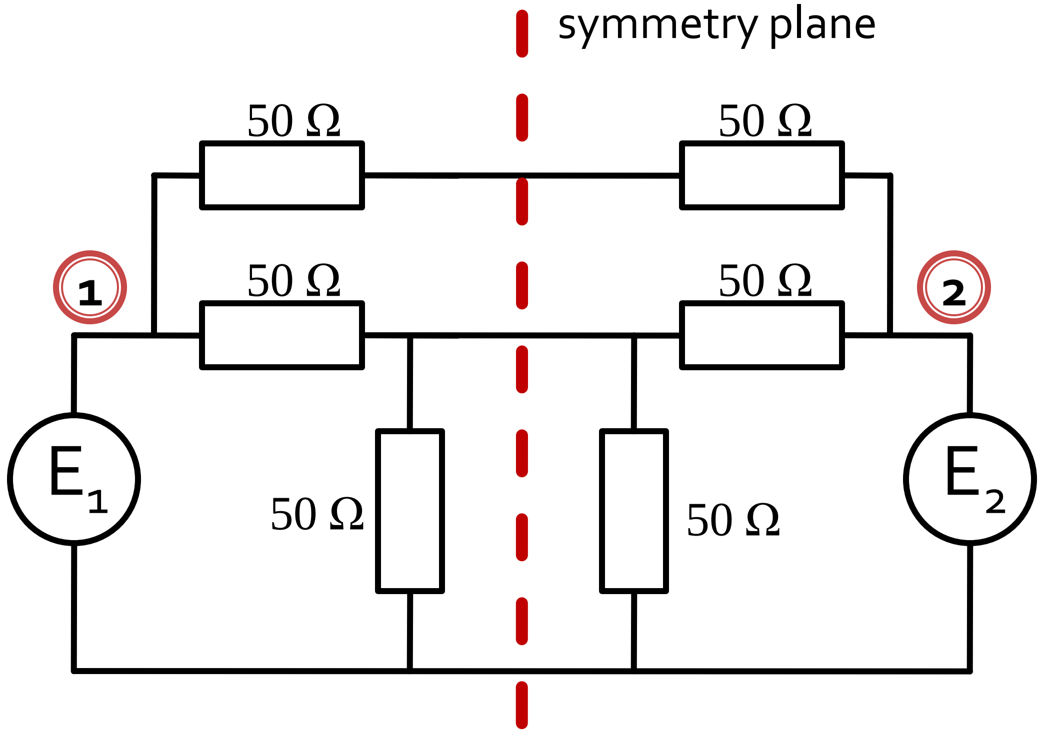
$$R_{ech} = 100\Omega \,||\, (50\Omega + 25\Omega \,||\, 50\Omega) =$$
$$= 100\Omega \,||\, (50\Omega + 16.67\Omega) = 100\Omega \,||\, 66.67\Omega = 40\Omega$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0.025S$$

Even/Odd Mode Analysis

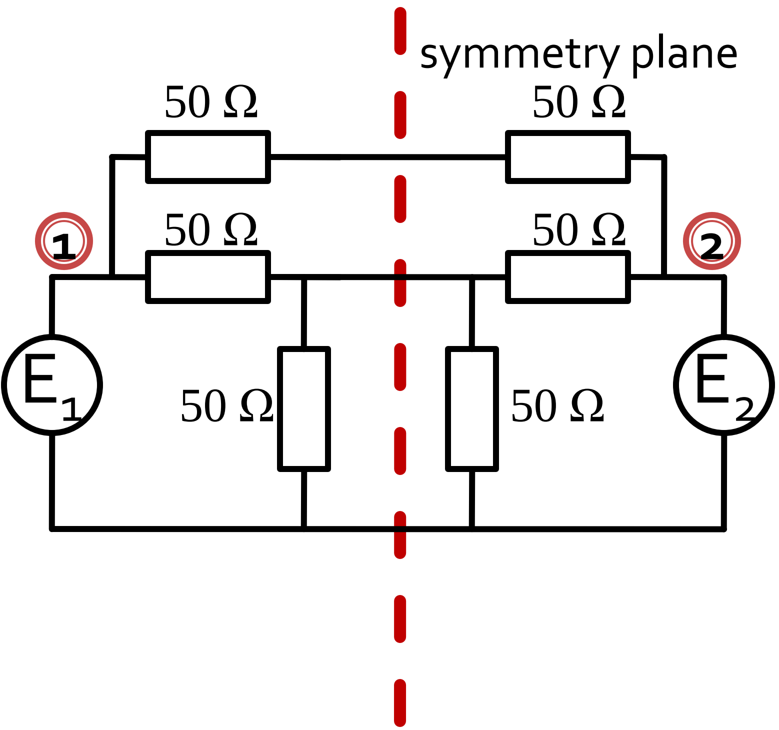
- Even/Odd mode analysis benefit from the existence of symmetry planes in the circuit
  - existing or
  - created (forced)

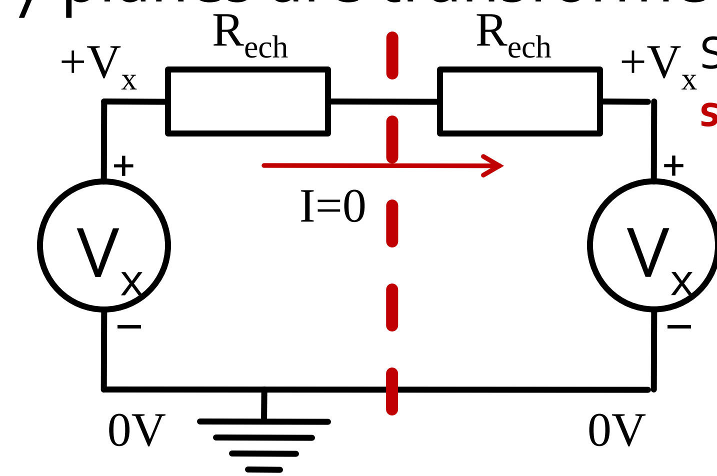




Even/Odd Mode Analysis

- when exciting the ports with symmetric/anti-symmetric sources the symmetry planes are transformed into:
  - open circuit
  - virtual ground

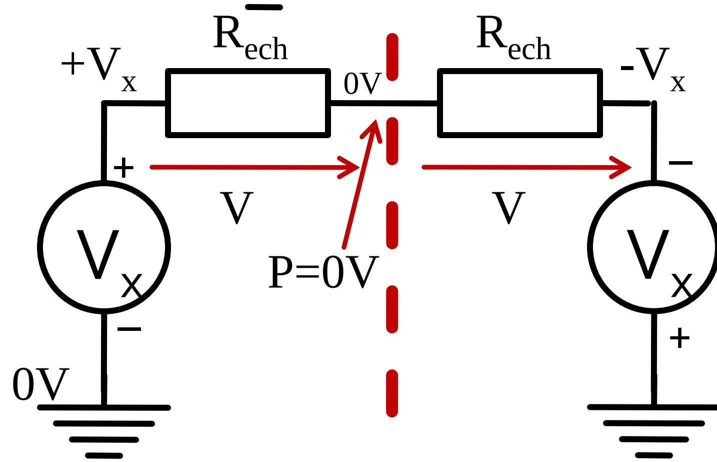




Sources have the **same** polarity

$$I = 0, \forall V_x$$

symmetry plane
open circuit



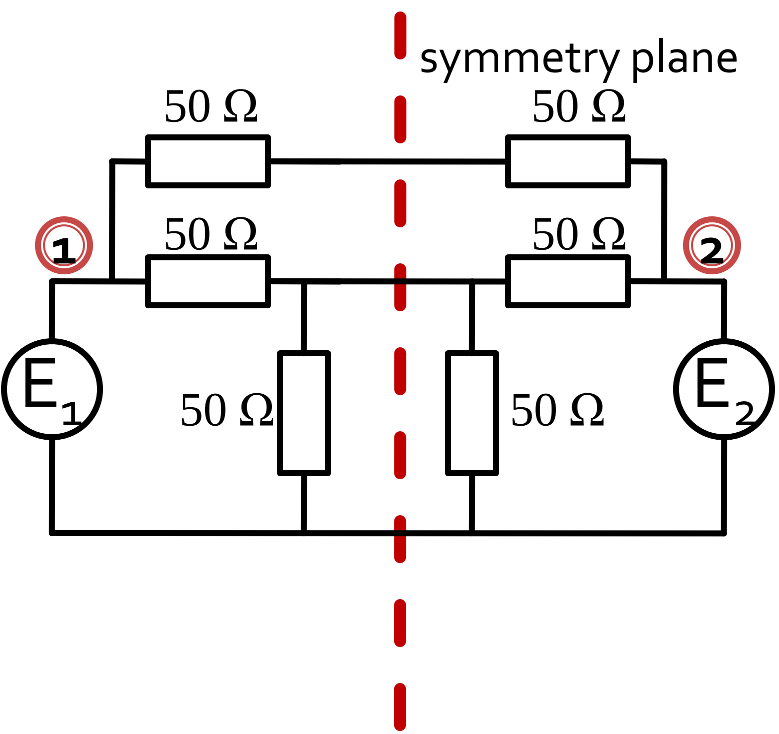
Sources have **opposite** polarities

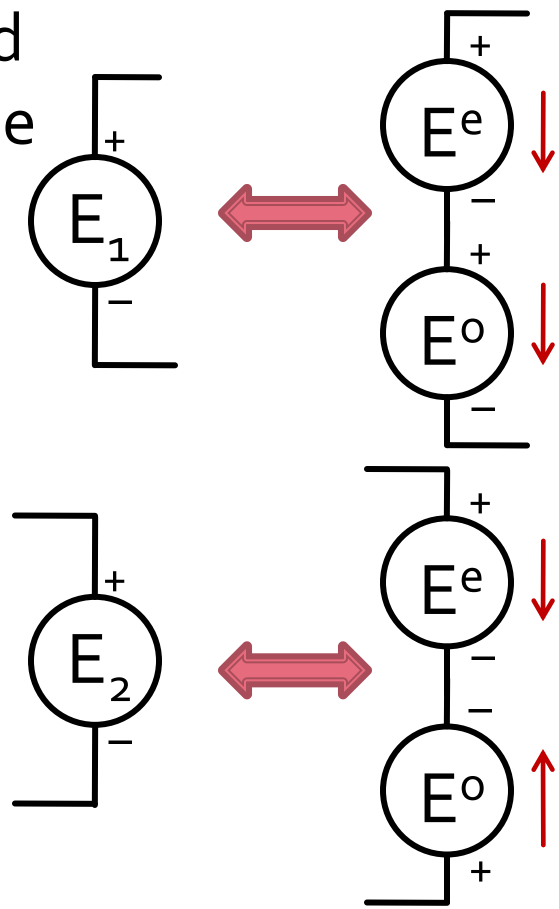
$$P = 0, \forall V_x$$

symmetry plane
virtual ground

Even/Odd Mode Analysis

- the combination of any two sources is equivalent for linear circuits with the superposition of:
  - a symmetric source and
  - a anti-symmetric source





$$E_1 = E^e + E^o$$

$$E_2 = E^e - E^o$$

$$E^e = \frac{E_1 + E_2}{2}$$

$$E^o = \frac{E_1 - E_2}{2}$$

Even/Odd Mode Analysis

- In linear circuits the **superposition principle** is always true
  - the response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually

Response (Source1 + Source2) =

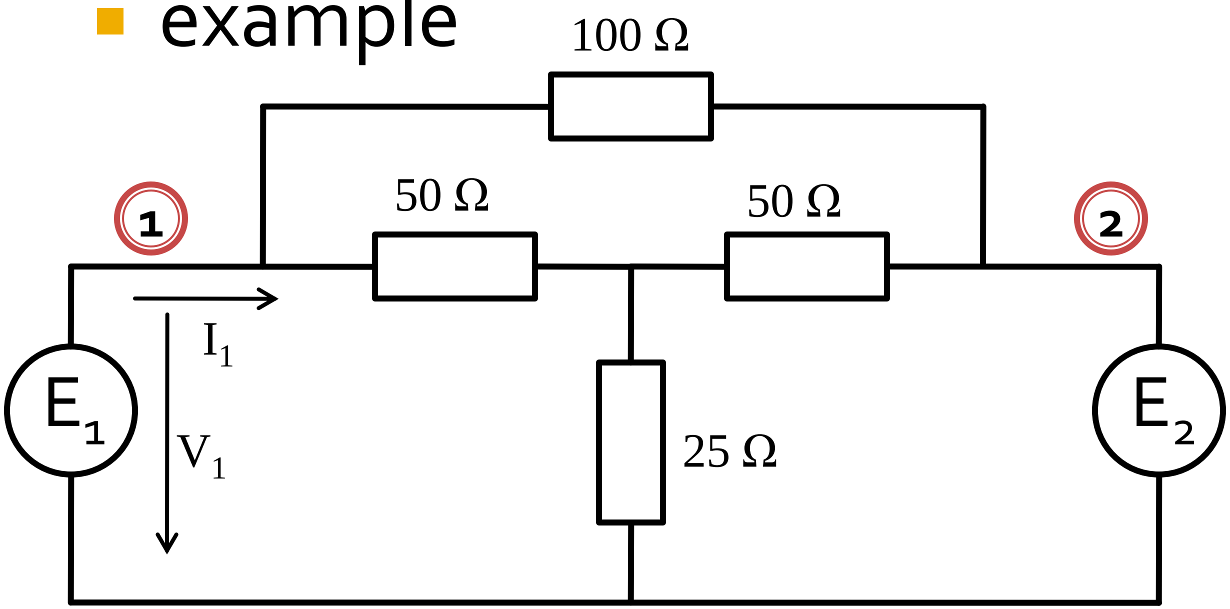
= Response (Source1) + Response (Source2)

Response(ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

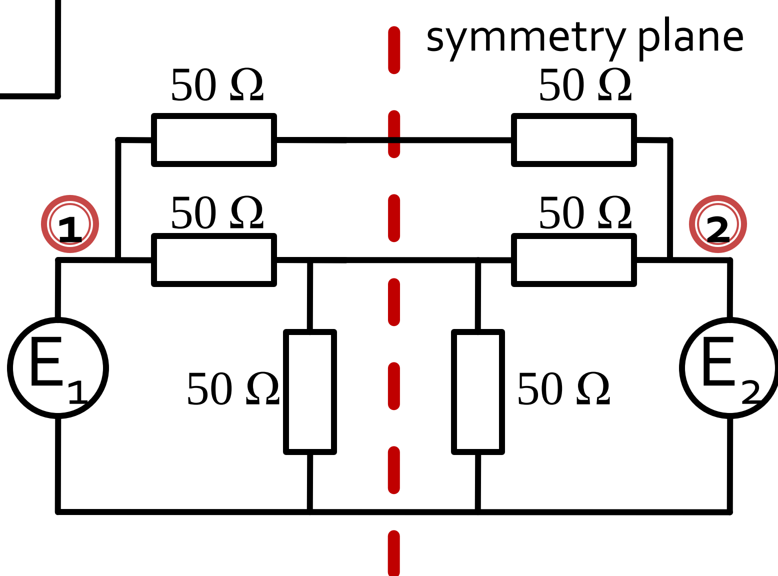
Even/Odd Mode Analysis

- example



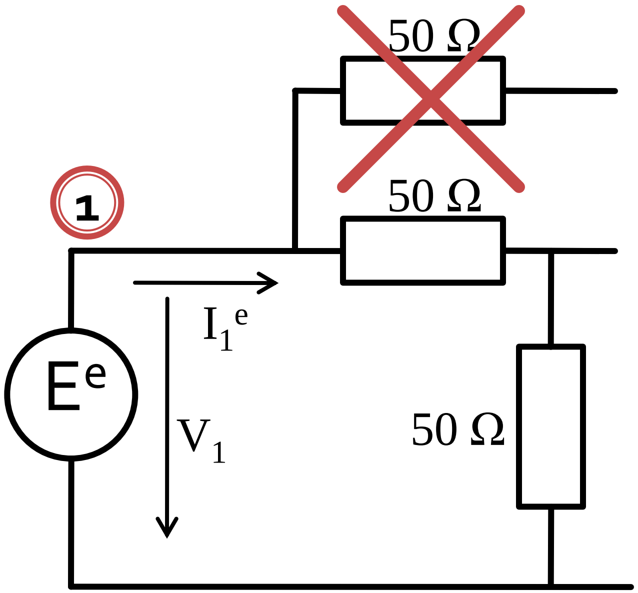
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

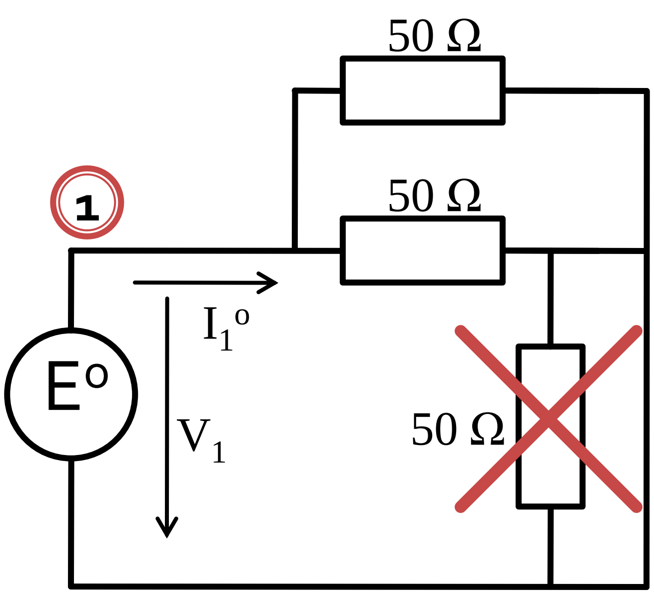
$$V_2 \equiv E_2 = 0 \Rightarrow \quad E^e = \frac{E_1}{2} \qquad E^o = \frac{E_1}{2}$$



Even/Odd Mode Analysis

- Even/Odd mode analysis





$$R_{ech}^{e} = 50\Omega + 50\Omega = 100\Omega$$

$$I_1^e = \frac{E^e}{R_{ech}^e} = \frac{E_1/2}{100\Omega} = \frac{E_1}{200\Omega}$$

EVEN → symmetry plane **open circuit**

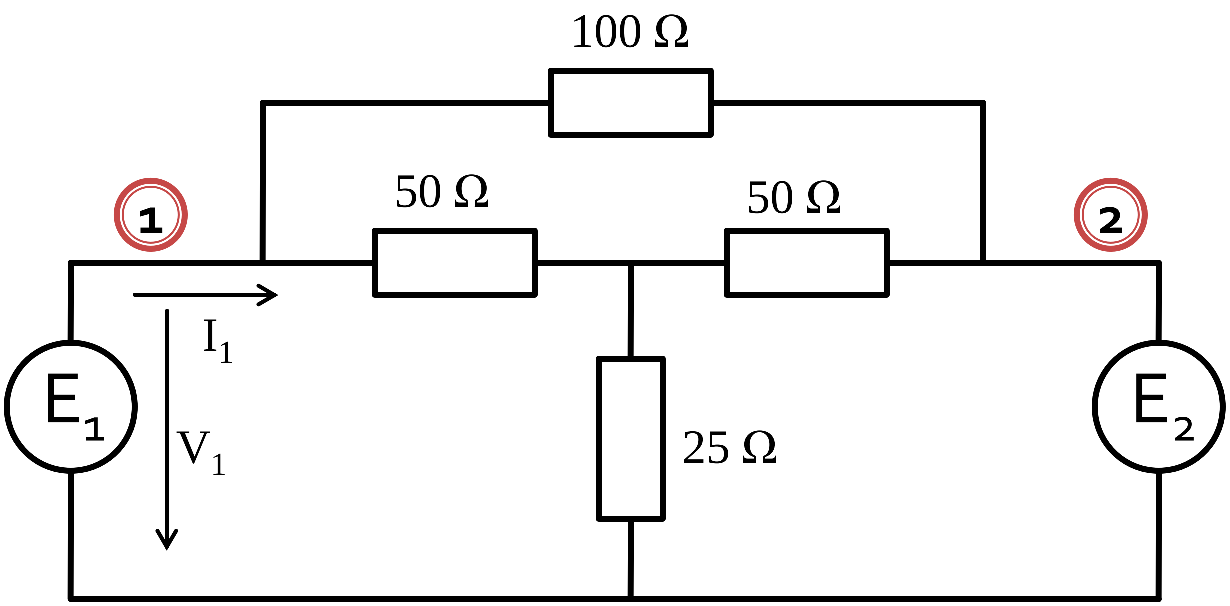
$$R_{ech}^{o} = 50\Omega \,||\, 50\Omega = 25\Omega$$

$$I_1^o = \frac{E^o}{R_{ech}^o} = \frac{E_1/2}{25\Omega} = \frac{E_1}{50\Omega}$$

ODD → symmetry plane **virtual ground**

Even/Odd Mode Analysis

- superposition principle



$$I_1 = I_1^e + I_1^o$$

$$V_1 = V_1^e + V_1^o$$

$$I_1 = I_1^e + I_1^o = \frac{E_1}{200\Omega} + \frac{E_1}{50\Omega} = \frac{E_1}{40\Omega}$$

$$V_1 = V_1^e + V_1^o = E_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{40\Omega} = 0.025S$$

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

Power dividers and directional couplers

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~

Introduction

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

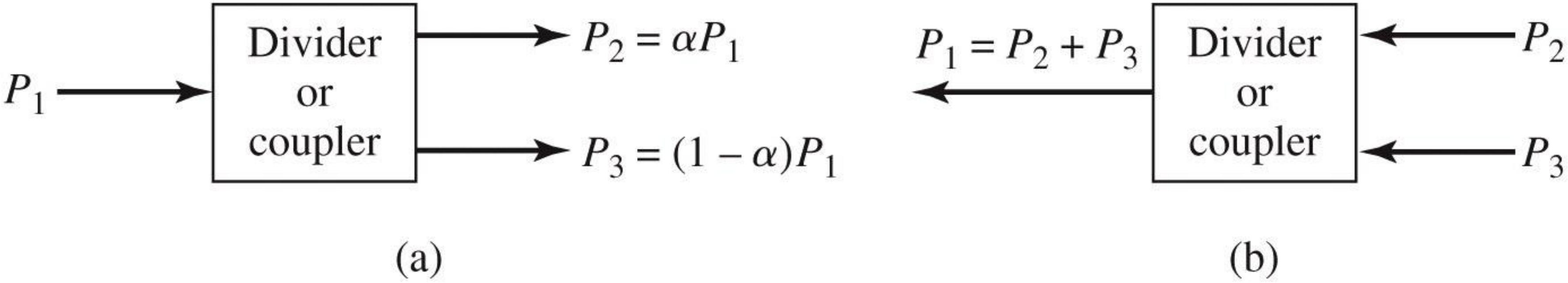
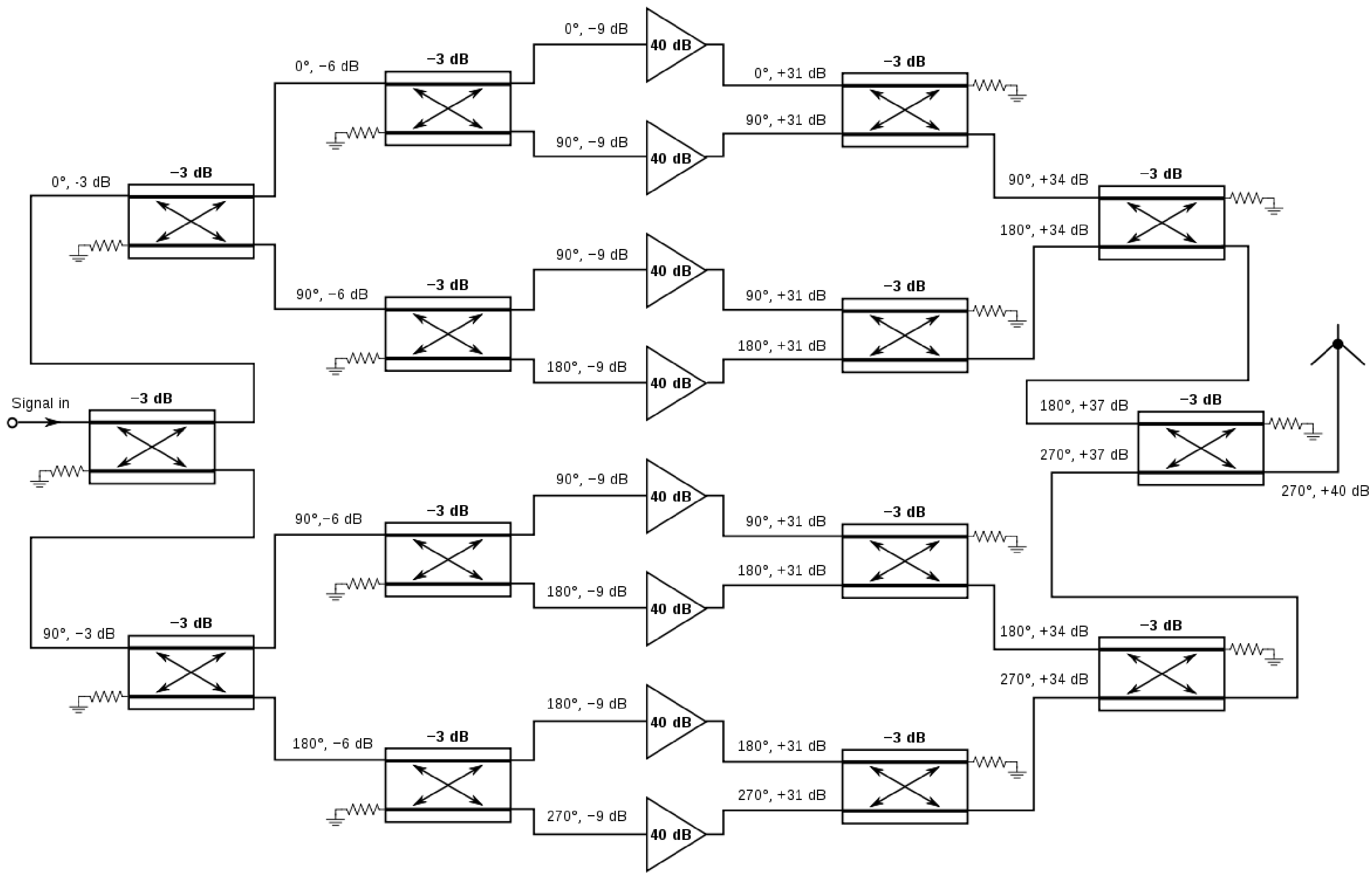


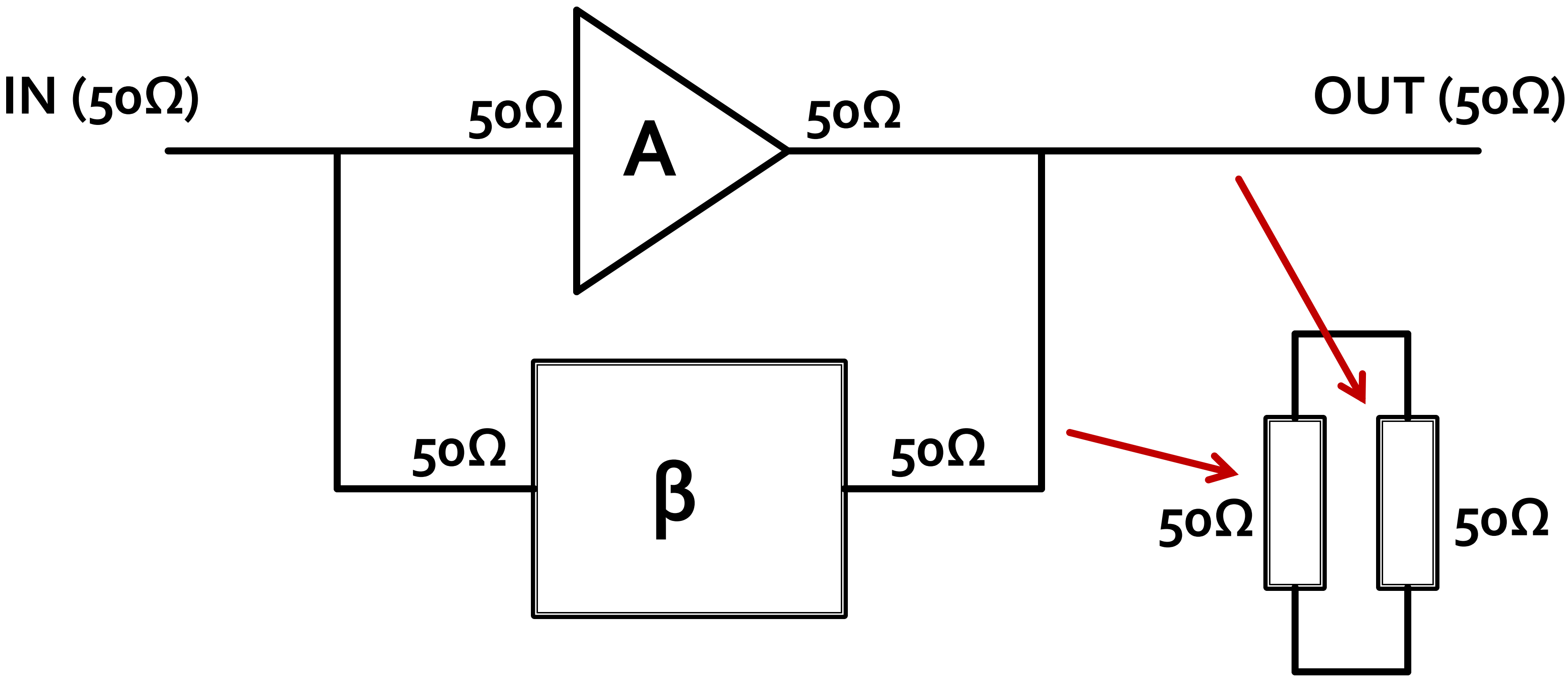
Figure 7.1

Balanced amplifiers



Matching

- feedback amplifier



Three-Port Networks

- also known as T-Junctions
- characterized by a 3x3 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Three-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Three-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^{*}\cdot[S]^{t}=[1] \qquad \sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=\delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{ki}^{*}=1 \qquad \sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=0, \forall i\neq j$$

Three-Port Networks

- lossless network

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{ki}^{*}=1$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=0, \forall i\neq j$$

- 6 equations / 3 unknowns

$$|S_{12}|^2+|S_{13}|^2=1 \qquad S_{13}^{*}S_{23}=0$$

$$|S_{12}|^2+|S_{23}|^2=1 \qquad S_{12}^{*}S_{13}=0$$

$$|S_{13}|^2+|S_{23}|^2=1 \qquad S_{23}^{*}S_{12}=0$$

- no solution is possible

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - lossless
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Nonreciprocal Three-Port Networks

- usually containing anisotropic materials, ferrites
- **nonreciprocal**, but matched at all ports and lossless $S_{ij} \neq S_{ji}$
- S matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- 6 equations / 6 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad S_{21}^* S_{23} = 0$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

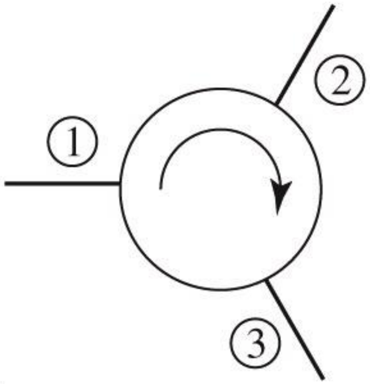
Nonreciprocal Three-Port Networks

- two possible solutions
- circulators
  - clockwise circulation

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



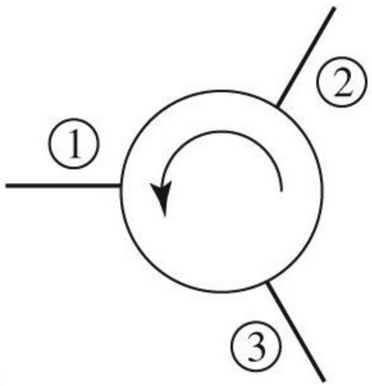
(a)

  - counterclockwise circulation

$$S_{21} = S_{32} = S_{13} = 0$$

$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

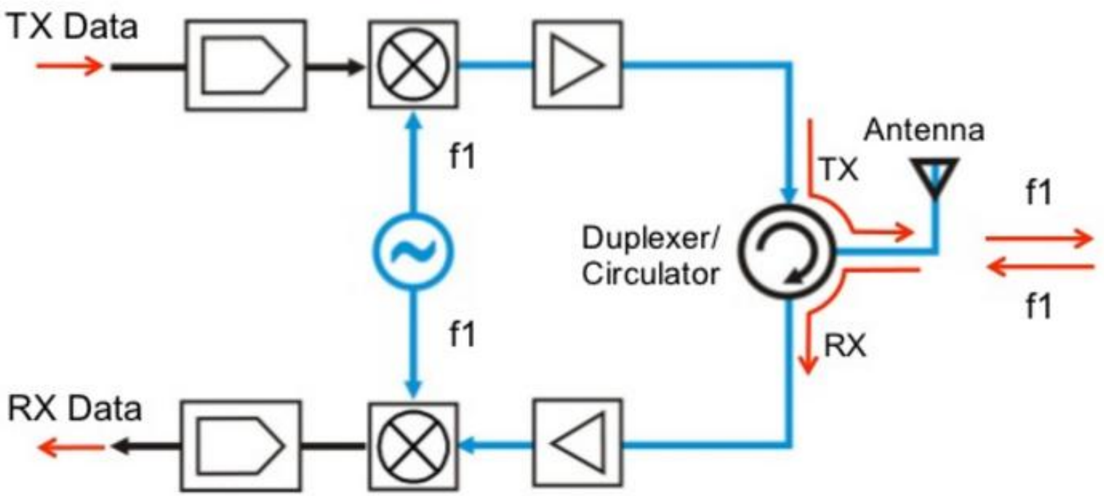
$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

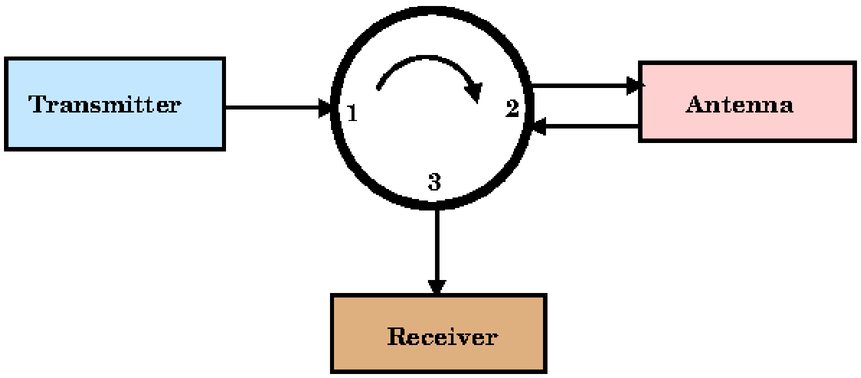


(b)

Nonreciprocal Three-Port Networks

- circulator often found in duplexer





Mismatched Three-Port Networks

- A lossless and reciprocal three-port network can be matched only on two ports, eg. 1 and 2:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}^{*}S_{23}=0$$

$$S_{12}^{*}S_{13}+S_{23}^{*}S_{33}=0$$

$$S_{23}^{*}S_{12}+S_{33}^{*}S_{13}=0$$

$$|S_{12}|^2+|S_{13}|^2=1$$

$$|S_{12}|^2+|S_{23}|^2=1$$

$$|S_{13}|^2+|S_{23}|^2+|S_{33}|^2=1$$

$$S_{13}=S_{23}=0$$

$$|S_{13}|=|S_{23}|$$

$$|S_{12}|=|S_{33}|=1$$

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network

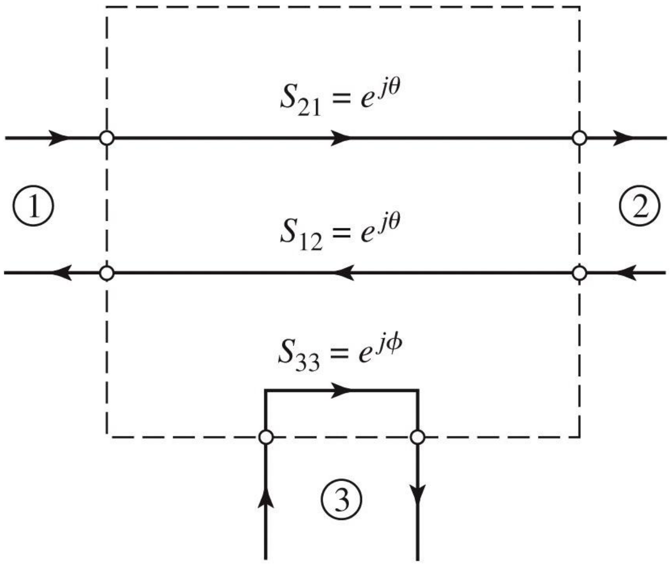
$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}=S_{23}=0 \qquad |S_{12}|=|S_{33}|=1$$

$$S_{12}=e^{j\theta}$$

$$S_{33}=e^{j\varphi}$$

$$[S]=\begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



- A lossless and reciprocal three-port network **degenerates** into two separate components:
  - a matched two-port **line**
  - a totally **mismatched one-port**:

Four-Port Networks

- characterized by a 4x4 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power “loss”

Four-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0, S_{44}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Four-Port Networks

$$\begin{aligned} S_{13}^* \cdot S_{23} + S_{14}^* \cdot S_{24} &= 0 \quad / \cdot S_{24}^* \\ S_{14}^* \cdot S_{13} + S_{24}^* \cdot S_{23} &= 0 \quad / \cdot S_{13}^* \end{aligned}$$

$$S_{14}^* \cdot \left(\left|S_{13}\right|^2 - \left|S_{24}\right|^2 \right) = 0$$

$$\begin{aligned} S_{12}^* \cdot S_{23} + S_{14}^* \cdot S_{34} &= 0 \quad / \cdot S_{12} \\ S_{14}^* \cdot S_{12} + S_{34}^* \cdot S_{23} &= 0 \quad / \cdot S_{34}^* \end{aligned}$$

$$S_{23} \cdot \left(\left|S_{12}\right|^2 - \left|S_{34}\right|^2 \right) = 0$$

- one solution: $S_{14} = S_{23} = 0$
- resulting coupler is **directional**

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$\left|S_{12}\right|^2 + \left|S_{13}\right|^2 = 1$$

$$\left|S_{12}\right|^2 + \left|S_{24}\right|^2 = 1$$

$$\left|S_{13}\right|^2 + \left|S_{34}\right|^2 = 1$$

$$\left|S_{24}\right|^2 + \left|S_{34}\right|^2 = 1$$

$$\left|S_{13}\right| = \left|S_{24}\right|$$

$$\left|S_{12}\right| = \left|S_{34}\right|$$

Four-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}| = |S_{34}| = \alpha \qquad |S_{13}| = |S_{24}| = \beta$$

β – voltage coupling coefficient

- We can choose the phase reference

$$S_{12} = S_{34} = \alpha \qquad S_{13} = \beta \cdot e^{j\theta} \qquad S_{24} = \beta \cdot e^{j\phi}$$

$$S_{12}^{*} \cdot S_{13} + S_{24}^{*} \cdot S_{34} = 0 \quad \rightarrow \quad \theta + \phi = \pi \pm 2 \cdot n \cdot \pi$$

$$|S_{12}|^2 + |S_{24}|^2 = 1 \quad \rightarrow \quad \alpha^2 + \beta^2 = 1$$

- The other possible solution for previous equations offer either essentially the same result (with a different phase reference) or the degenerate case (2 separate two port networks side by side)

$$S_{14}^{*} \cdot \left(|S_{13}|^2 - |S_{24}|^2\right) = 0 \qquad S_{23} \cdot \left(|S_{12}|^2 - |S_{34}|^2\right) = 0$$

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler (90°) $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler (180°) $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Directional Coupler

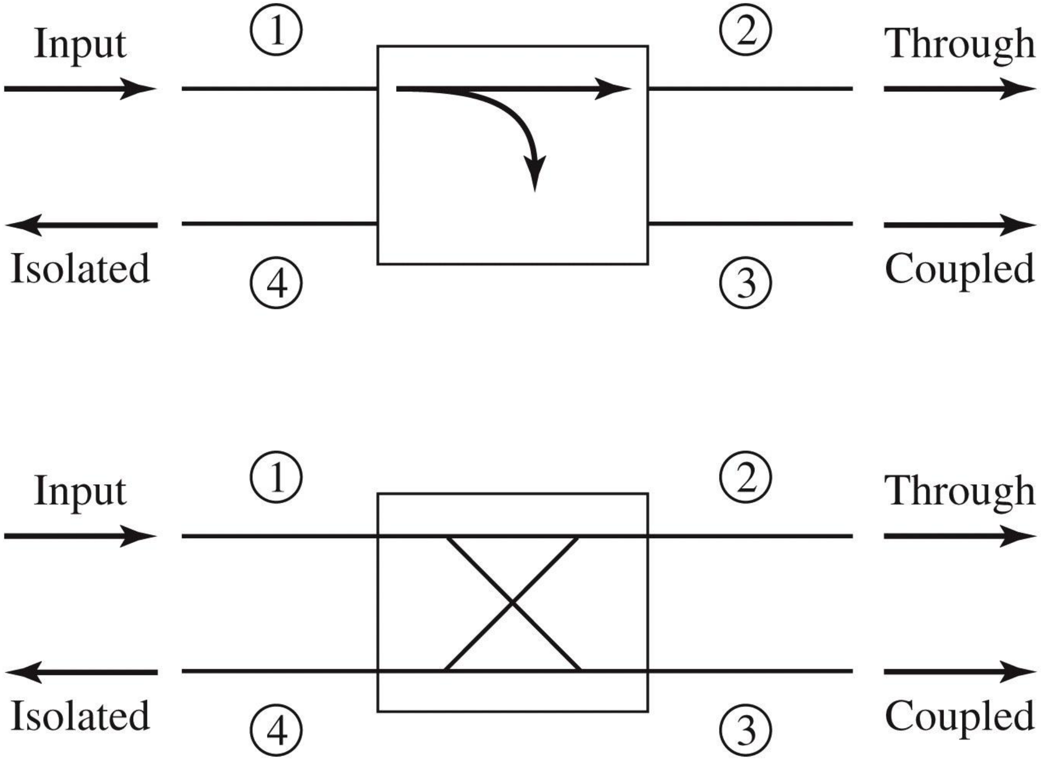


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20 \cdot \log(\beta)\,[\mathrm{dB}]$$

Directivity

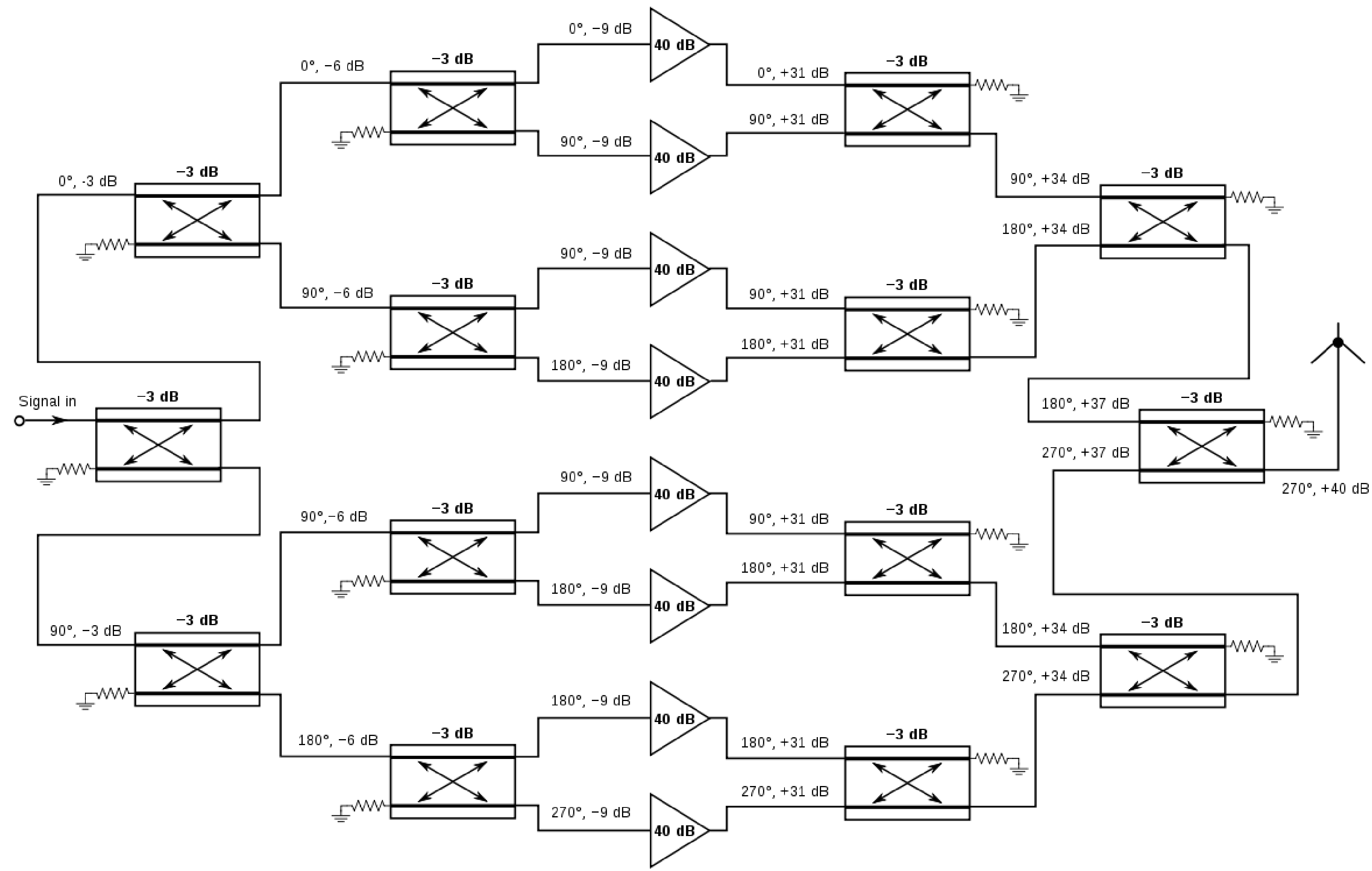
$$D = 10\log\frac{P_3}{P_4} = 20 \cdot \log\left(\frac{\beta}{|S_{14}|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20 \cdot \log|S_{14}|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \ [dB]$$

Balanced amplifiers



Power dividers

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - **lossless**
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Power division of the T-junction

- consists in splitting an input line into two separate output lines
- available in various technologies for the lines

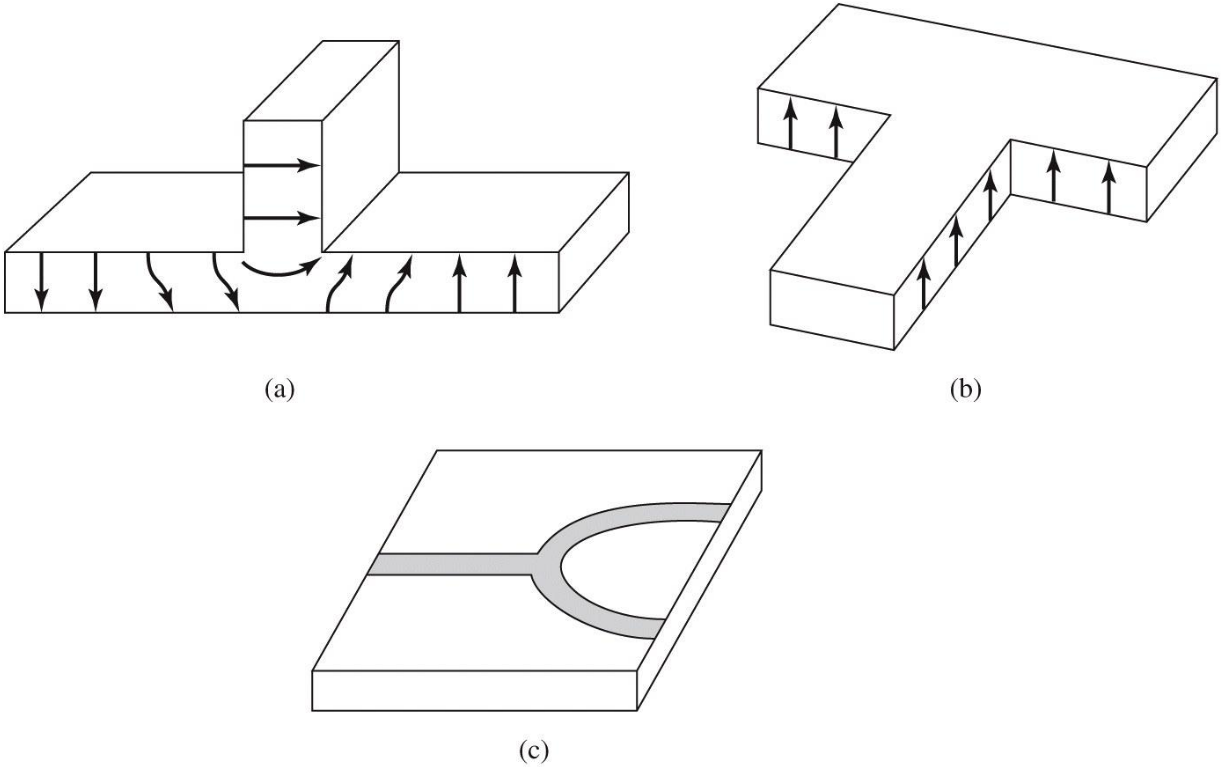


Figure 7.5

Power division of the T-junction

- if the lines are lossless, the network is reciprocal, so it cannot be matched at all ports simultaneously

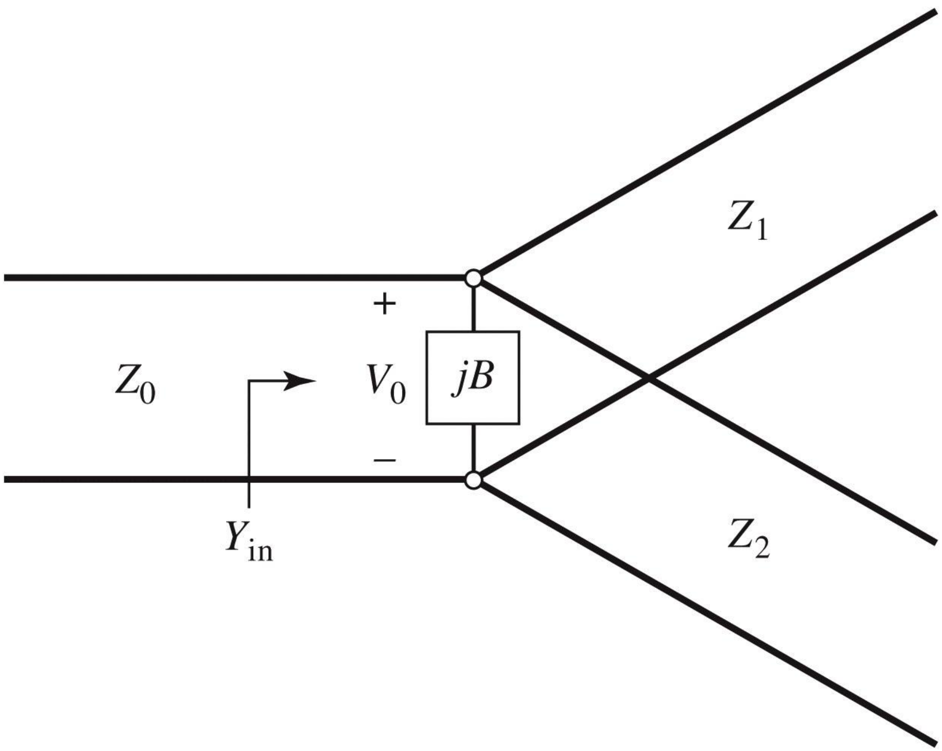


Figure 7.6

- there may be fringing fields and higher order modes associated with the discontinuity at such a junction
- the stored energy can be accounted for by a lumped susceptance: **B**
- Designing the power divider targets matching to the input line Z_o
  - outputs (unmatched, Z_1 and Z_2) can be, if needed, matched to Z_o ($\lambda/4$, binomial, Chebyshev)

Power division of the T-junction

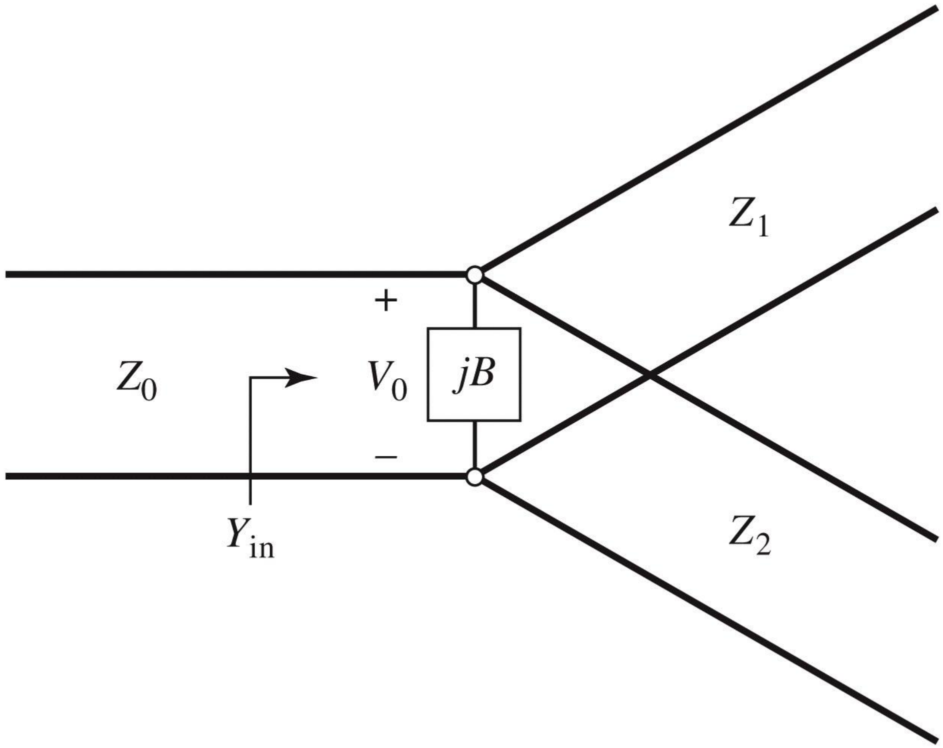


Figure 7.6

$$Y_{in} = j \cdot B + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

- If the transmission lines are assumed to be lossless, then the characteristic impedances are real
- the matching condition can be met only if **B ≅ 0** thus the matching condition is:

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

In practice, if **B** is not negligible, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel this susceptance, at least over a narrow frequency range.

Power division of the T-junction

- if V_o is the voltage at the junction, we can compute how the input power is divided between the two output lines

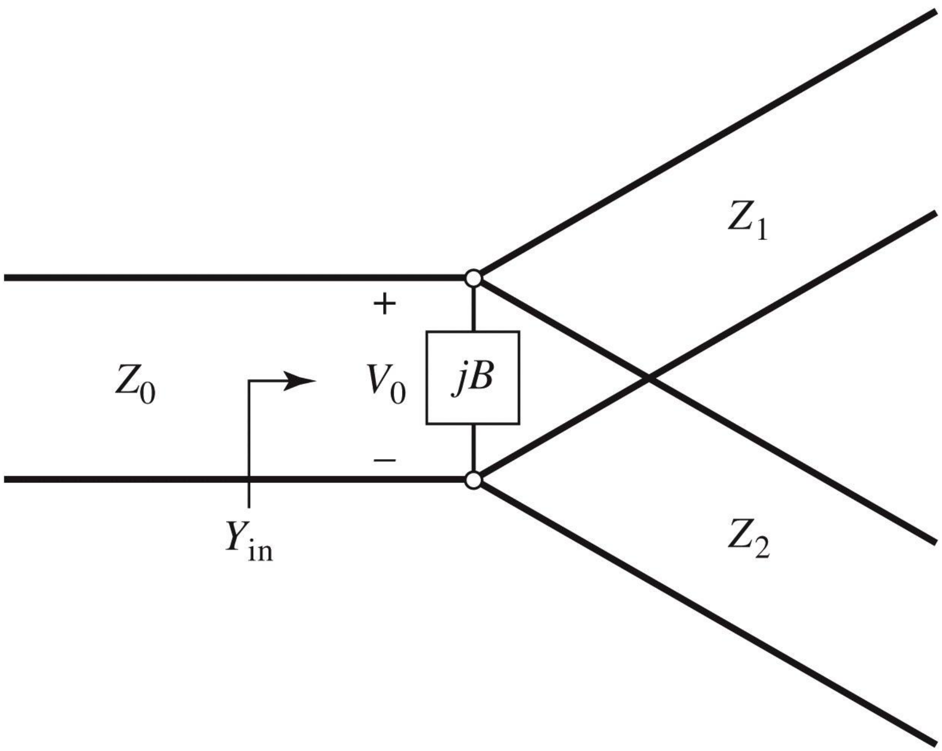


Figure 7.6

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1}$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2}$$

then:

$$P_{in} = P_1 + P_2$$ (lossless/input matching)

$$\frac{P_1}{P_2} = \frac{Z_2}{Z_1} = \alpha$$ (power division between the two output lines)

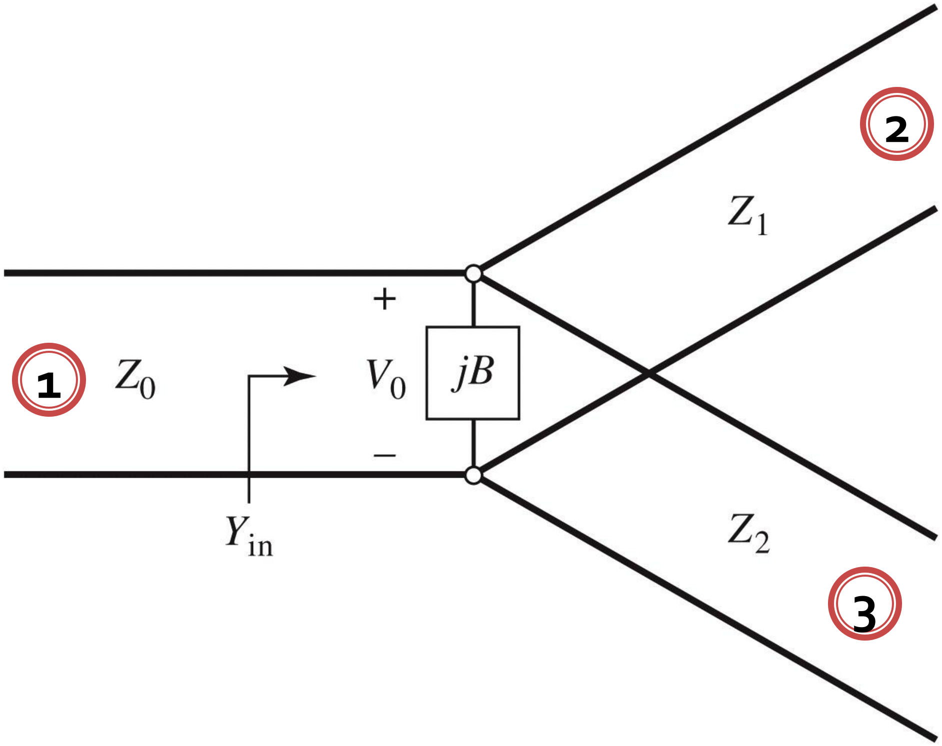
$$P_1 = P_{in} \cdot \frac{Z_2}{Z_1 + Z_2} \qquad P_2 = P_{in} \cdot \frac{Z_1}{Z_1 + Z_2}$$

$$P_1 = P_{in} \cdot \frac{\alpha}{1+\alpha} \qquad P_2 = P_{in} \cdot \frac{1}{1+\alpha}$$

$$Z_1 = Z_0 \cdot \left(1 + \frac{1}{\alpha}\right) \qquad Z_2 = Z_0 \cdot (1+\alpha)$$

Power division of the T-junction

- S matrix
  - lossless (unitary matrix)
  - reciprocal (symmetrical matrix)
  - input port is matched $S_{11} = 0$



$$P_2 = P_1 \cdot \frac{\alpha}{1+\alpha} \qquad S_{21} = S_{12} = \sqrt{\frac{\alpha}{1+\alpha}}$$

$$P_3 = P_1 \cdot \frac{1}{1+\alpha} \qquad S_{31} = S_{13} = \sqrt{\frac{1}{1+\alpha}}$$

the reflection coefficients seen looking into the output ports

$$S_{22} = \Gamma_1 = \frac{Z_0 \parallel Z_2 - Z_1}{Z_0 \parallel Z_2 + Z_1} = -\frac{1}{1+\alpha}$$

$$S_{33} = \Gamma_2 = \frac{Z_0 \parallel Z_1 - Z_2}{Z_0 \parallel Z_1 + Z_2} = -\frac{\alpha}{1+\alpha}$$

Figure 7.6

Power division of the T-junction

$$[S]=\begin{bmatrix} 0 & \sqrt{\dfrac{\alpha}{1+\alpha}} & \sqrt{\dfrac{1}{1+\alpha}} \\ \sqrt{\dfrac{\alpha}{1+\alpha}} & -\dfrac{1}{1+\alpha} & x \\ \sqrt{\dfrac{1}{1+\alpha}} & x & -\dfrac{\alpha}{1+\alpha} \end{bmatrix}$$

Unitary matrix, columns 1 and 2

$$0-\frac{1}{1+\alpha}\cdot\sqrt{\frac{\alpha}{1+\alpha}}+x\cdot\sqrt{\frac{1}{1+\alpha}}=0$$

$$S_{23}=S_{32}=\frac{\sqrt{\alpha}}{1+\alpha}$$

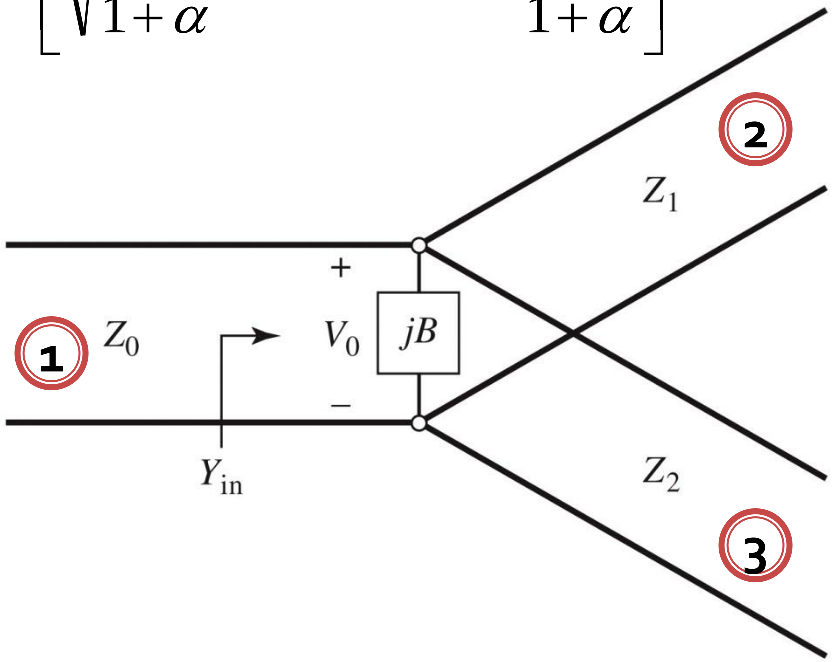


Figure 7.6

$$[S]=\begin{bmatrix} 0 & \sqrt{\dfrac{\alpha}{1+\alpha}} & \sqrt{\dfrac{1}{1+\alpha}} \\ \sqrt{\dfrac{\alpha}{1+\alpha}} & -\dfrac{1}{1+\alpha} & \dfrac{\sqrt{\alpha}}{1+\alpha} \\ \sqrt{\dfrac{1}{1+\alpha}} & \dfrac{\sqrt{\alpha}}{1+\alpha} & -\dfrac{\alpha}{1+\alpha} \end{bmatrix}$$

Power division of the T-junction

- 3dB divider
  - equal splitting of the power between the two outputs
  - $Z_1 = Z_2 = 2 \cdot Z_0$, $\alpha = 1$

$$[S] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

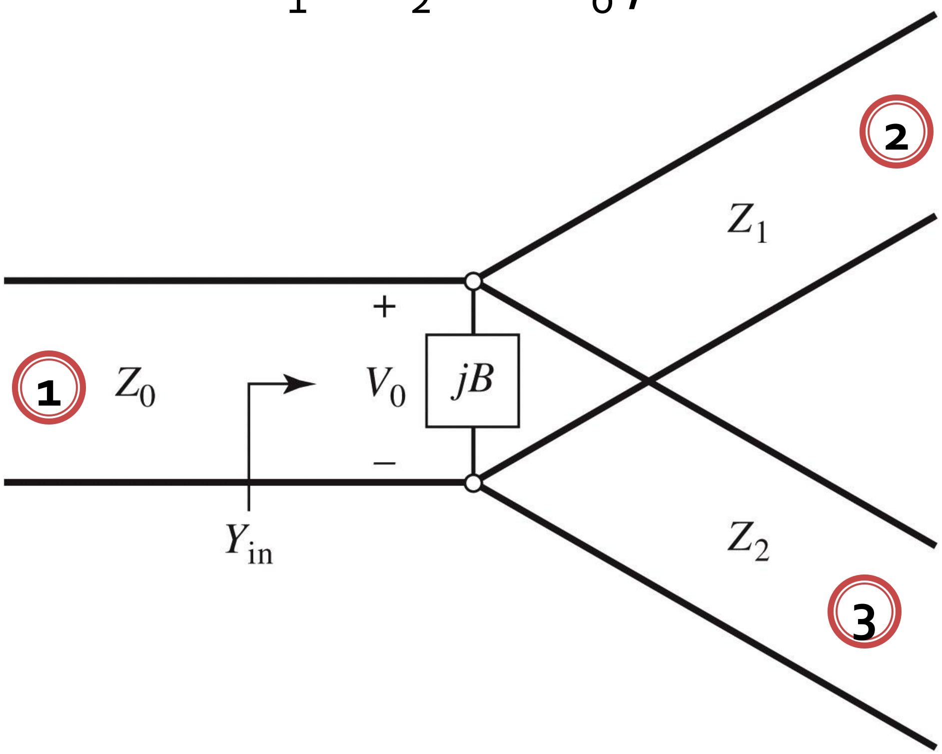


Figure 7.6

If we add $\lambda/4$ transformers to match outputs to Z_0 S matrix:

$$[S] = \begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{j}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Example

- Design a lossless T-junction divider with a 30Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 30Ω. (**Pozar** problem)

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0} \qquad \begin{cases} P_1 + P_2 = P_{in} \\ P_1 : P_2 = 3:1 \end{cases} \Rightarrow \begin{cases} P_1 = \frac{1}{4} \cdot P_{in} \\ P_2 = \frac{3}{4} \cdot P_{in} \end{cases}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1} = \frac{1}{4} \cdot P_{in} \qquad Z_1 = 4 \cdot Z_0 = 120\,\Omega$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2} = \frac{3}{4} \cdot P_{in} \qquad Z_2 = 4 \cdot Z_0 / 3 = 40\,\Omega$$

Input match check

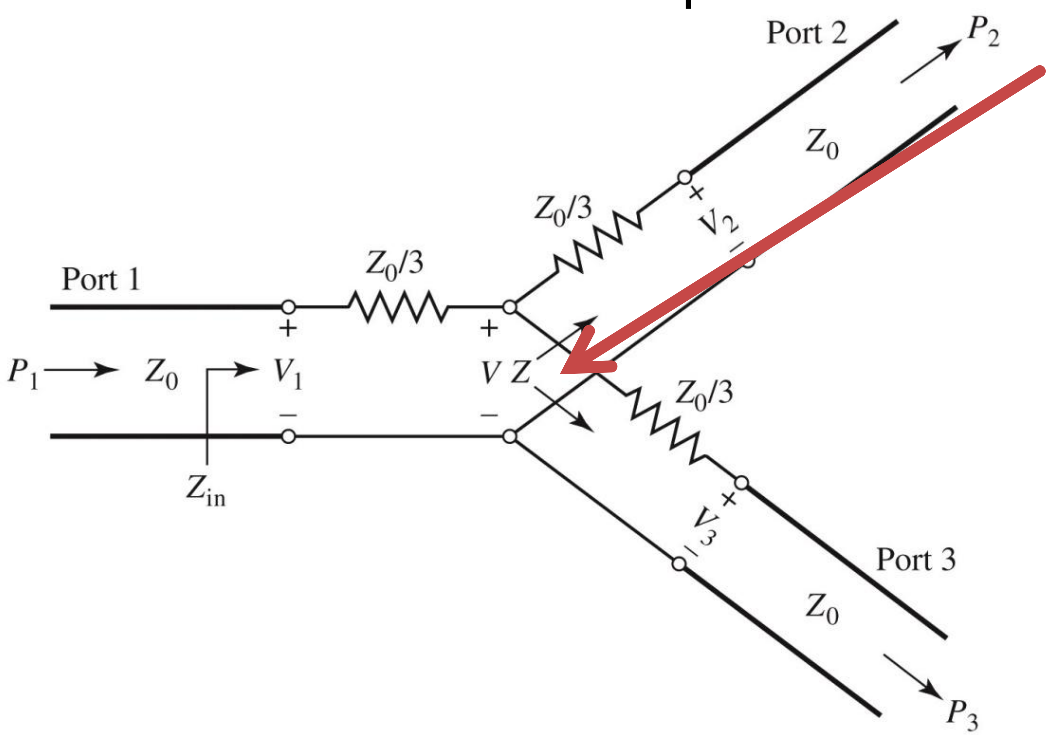
$$Z_{in} = 40\,\Omega \,\|\, 120\,\Omega = 30\,\Omega$$

quarter-wave transformers $Z_c^i = \sqrt{Z_i \cdot Z_L}$

$$Z_c^1 = \sqrt{Z_1 \cdot Z_L} = \sqrt{120\Omega \cdot 30\Omega} = 60\Omega \qquad Z_c^2 = \sqrt{Z_2 \cdot Z_L} = \sqrt{40\Omega \cdot 30\Omega} = 34.64\Omega$$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports



The impedance Z, seen looking into the Zo/3 resistor followed by a terminated output line:

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

The input line will be terminated with a Zo/3 resistor in series with two such lines Z in parallel

$$Z_{in} = \frac{Z_0}{3} + \frac{1}{2} \cdot \frac{4Z_0}{3} = Z_0$$

so it will be matched: $S_{11} = 0$

from symmetry: $S_{11} = S_{22} = S_{33} = 0$

Figure 7.7

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

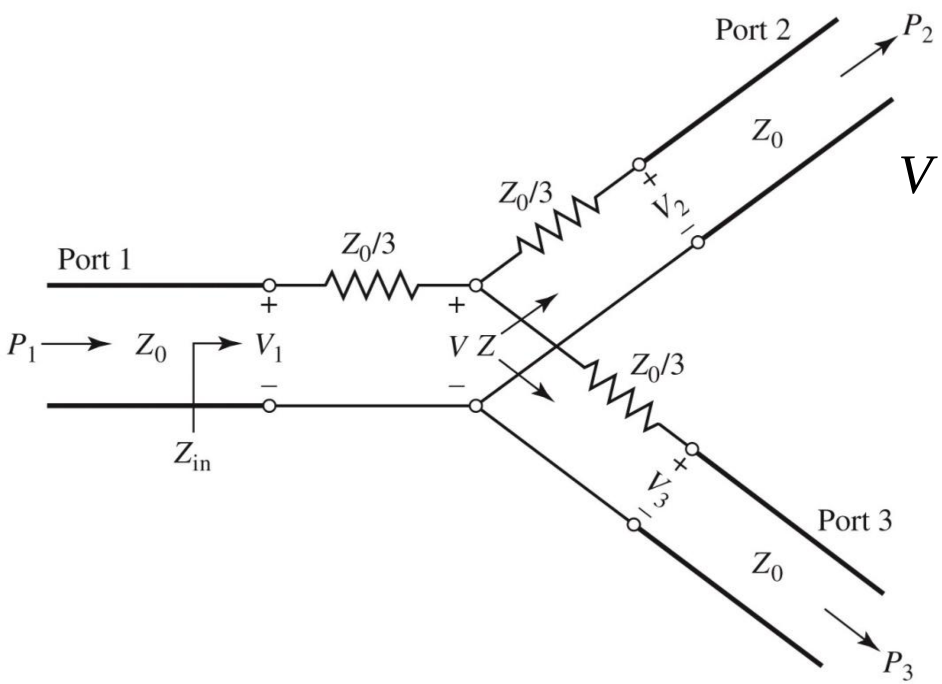


Figure 7.7

If the voltage at port 1 is V1, then by voltage division the voltage V at the junction is:

$$V = V_1 \cdot \frac{Z/2}{Z/2 + Z_0/3} = V_1 \cdot \frac{2Z_0/3}{2Z_0/3 + Z_0/3} = \frac{2}{3} \cdot V_1$$

The output voltages are, again by voltage division :

$$V_2 = V_3 = V \cdot \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4} \cdot V = \frac{1}{2} \cdot V_1$$

$$S_{21} = S_{31} = \frac{1}{2}$$

from symmetry: $S_{21} = S_{31} = S_{23} = \frac{1}{2}$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal (S matrix is symmetrical) $S_{21} = S_{31} = S_{23} = \frac{1}{2}$
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

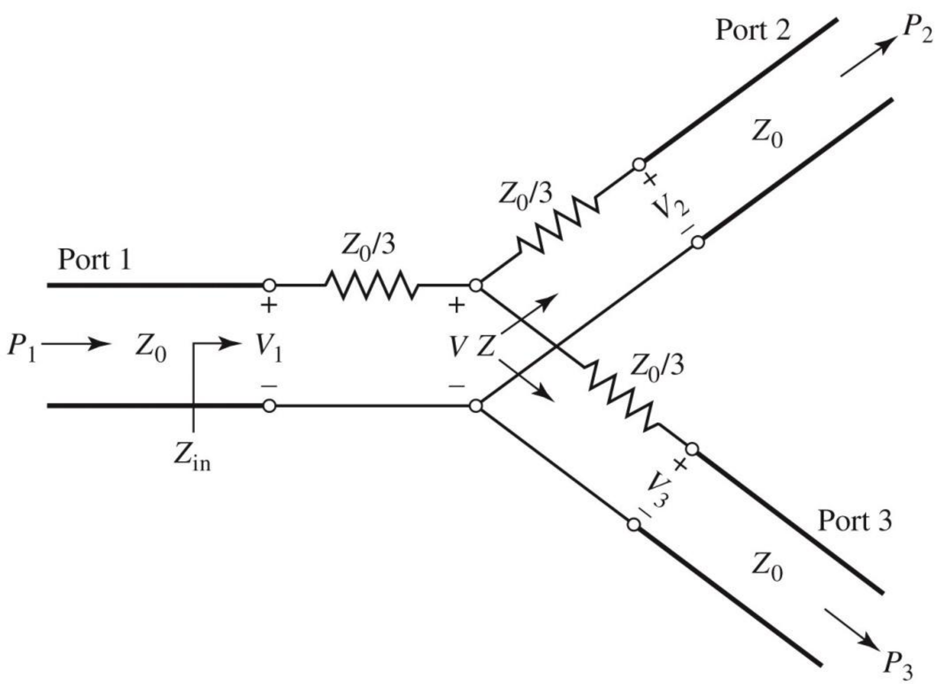


Figure 7.7

S matrix: $$[S] = \frac{1}{2} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

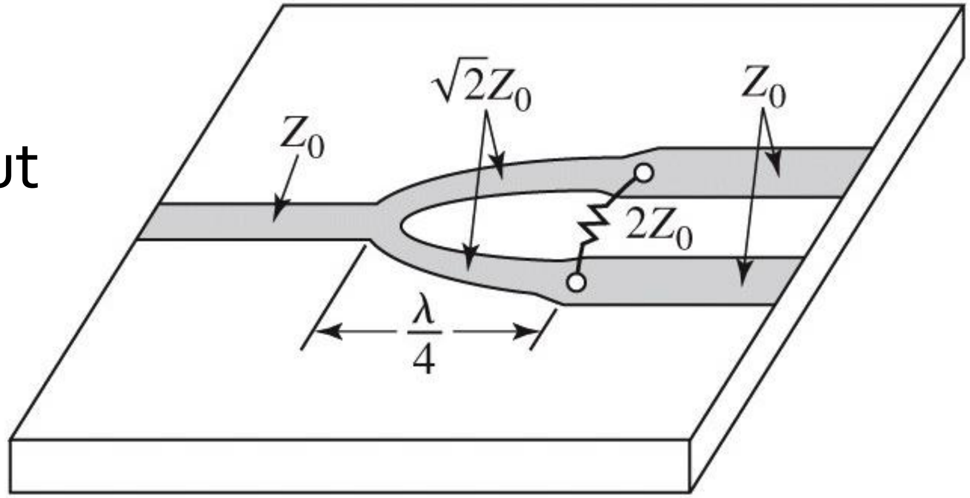
Powers: $$P_{in} = \frac{1}{2} \cdot \frac{V_1^2}{Z_0}$$

$$P_2 = P_3 = \frac{1}{2} \cdot \frac{(1/2V_1)^2}{Z_0} = \frac{1}{8} \cdot \frac{V_1^2}{Z_0} = \frac{1}{4} \cdot P_{in}$$

Half of the supplied power is dissipated in the 3 resistors. The output powers are 6 dB below the input power level

The Wilkinson power divider

- Previous power dividers suffer from a major drawback, there is not isolation between the two output ports $S_{23} = S_{32} \neq 0$
  - this requirement is important in some applications
- The Wilkinson power divider solves this problem
  - it also has the useful property of appearing **lossless** when the output ports are matched
  - only reflected power from the output ports is dissipated



The Wilkinson power divider

- one input line
- two λ/4 transformers
- one resistor between the output lines

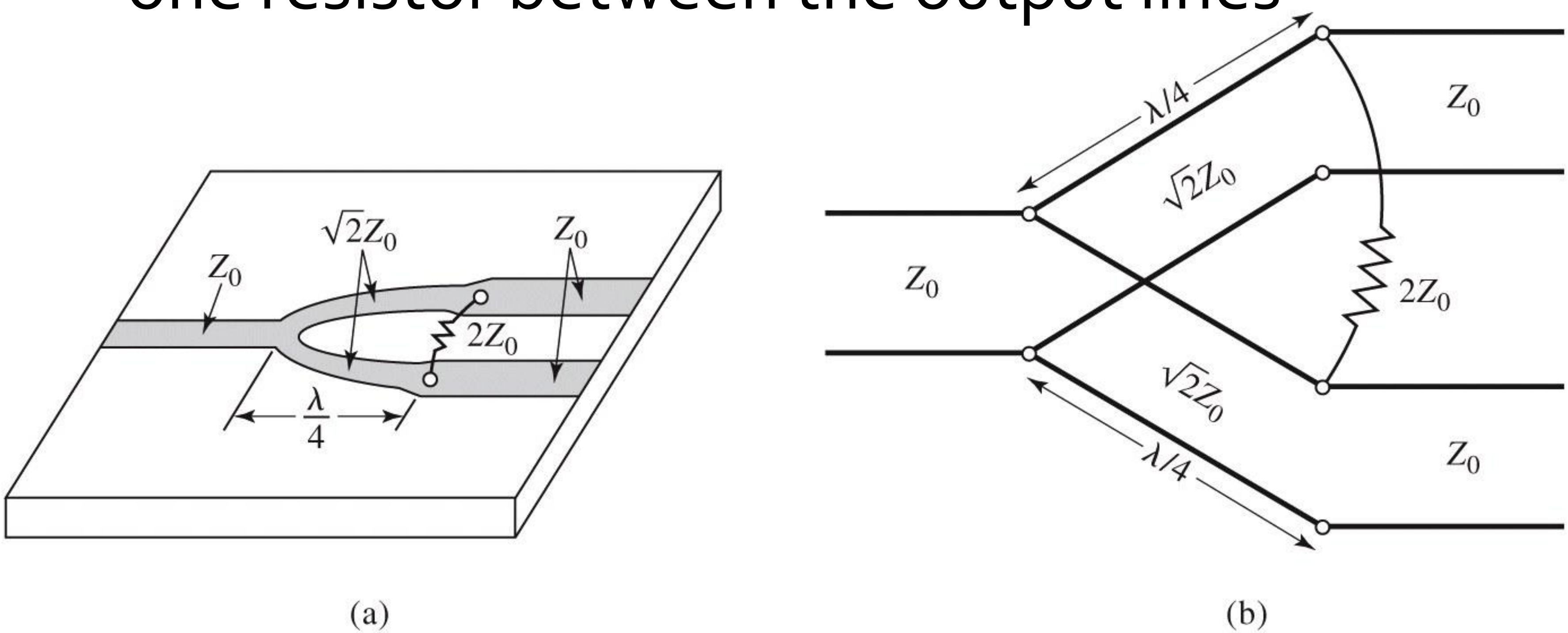


Figure 7.8

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

The Wilkinson power divider

- the circuit in normalized and symmetric form

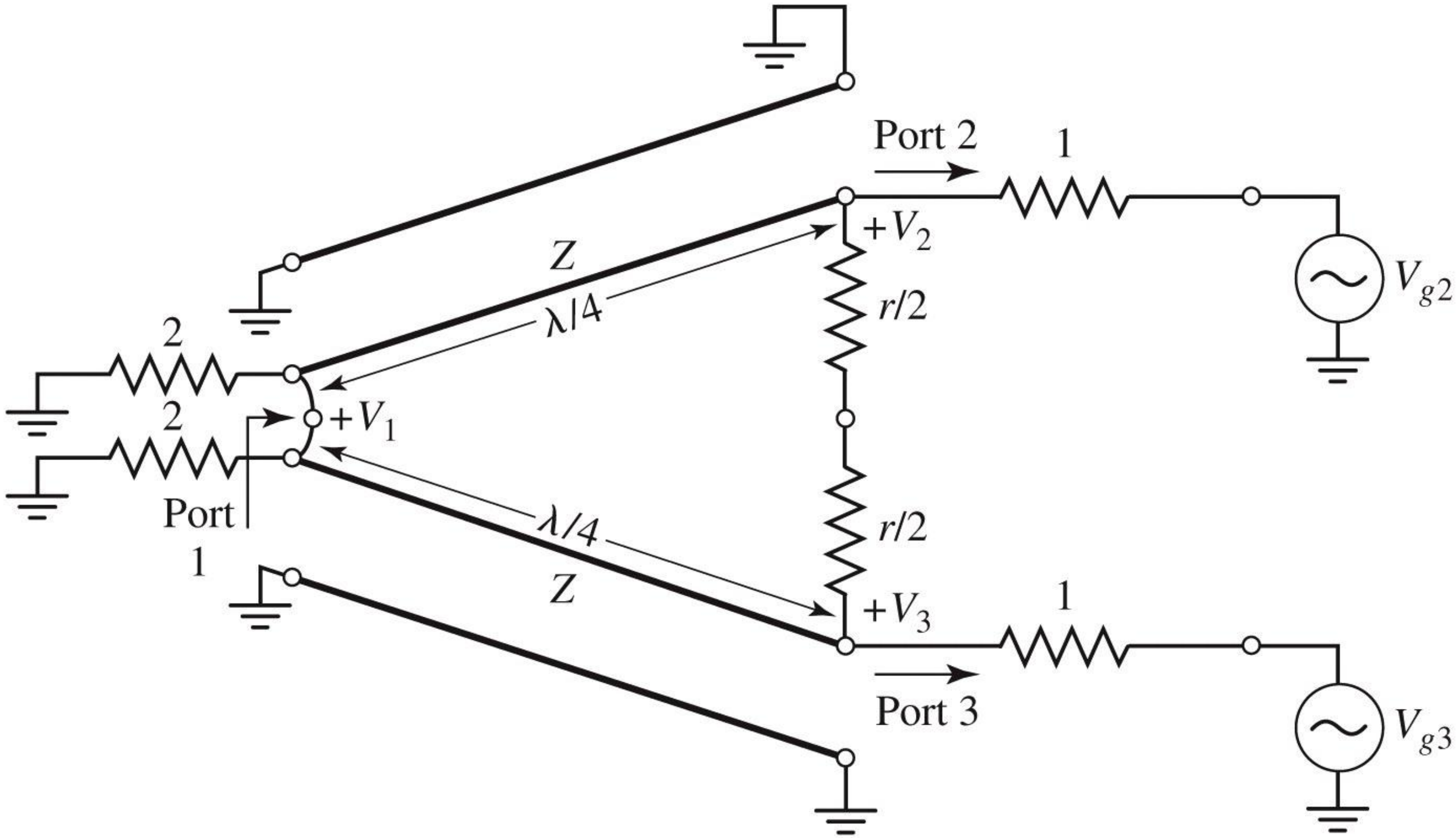
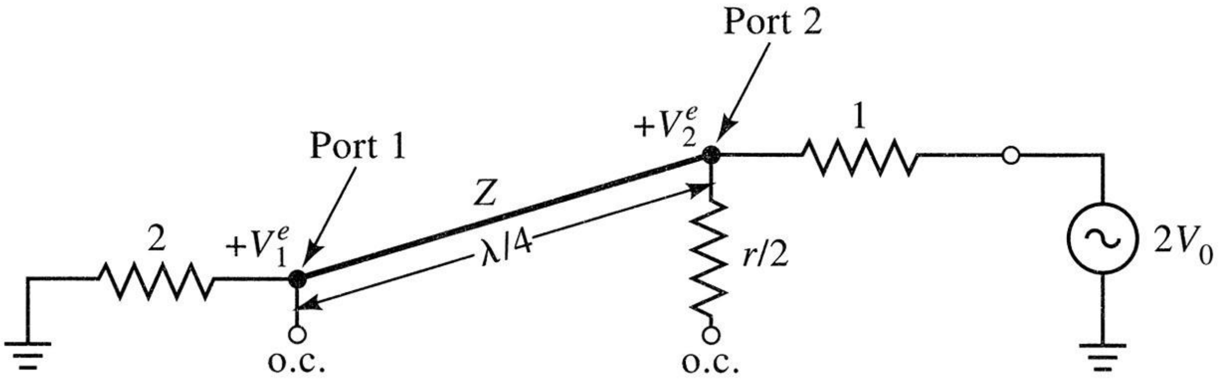


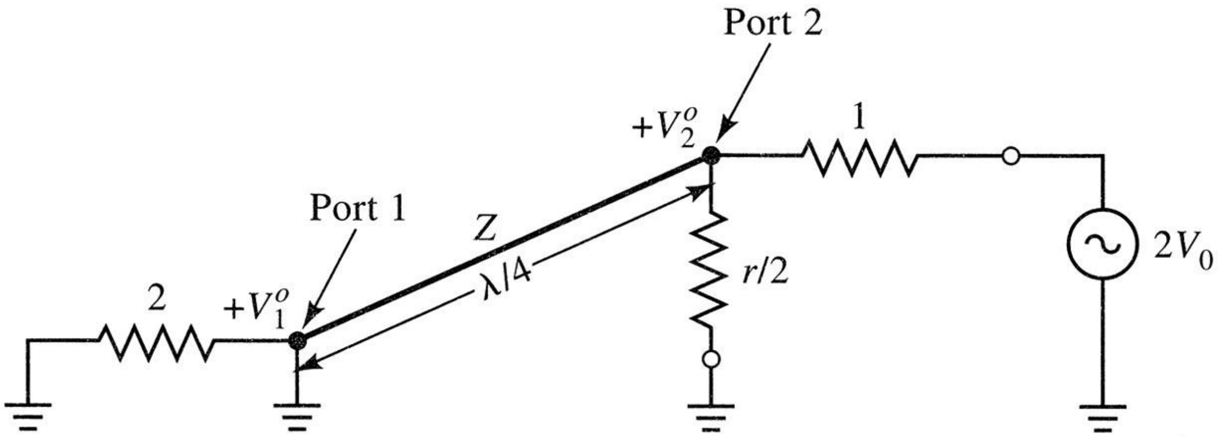
Figure 7.9

The Wilkinson power divider

- Even/Odd Mode Analysis



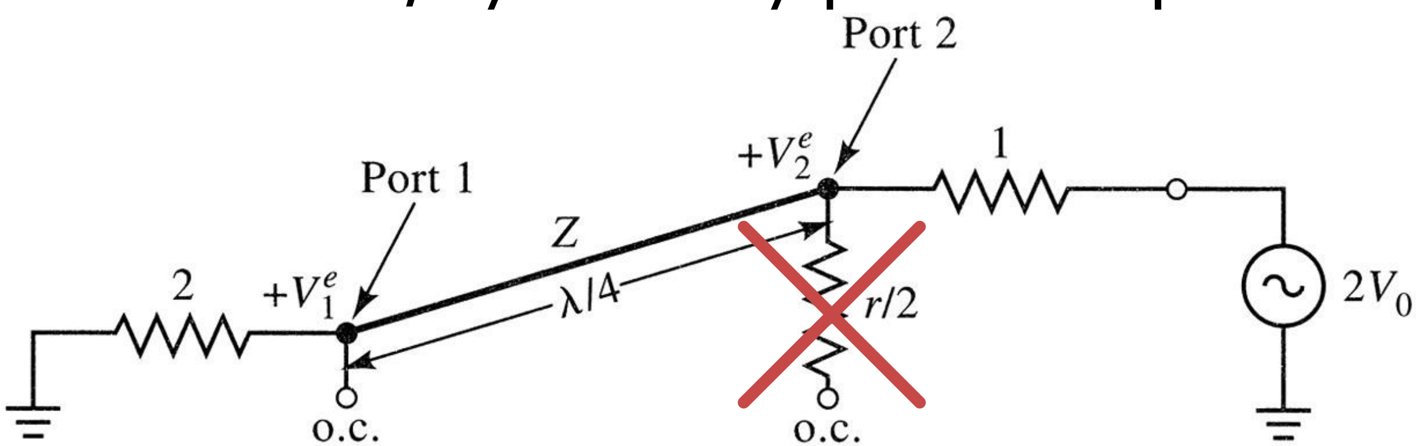
(a)



(b)

The Wilkinson power divider

- even mode, symmetry plane is open circuit



looking into port 2, λ/4 transformer with 2 load $Z_{in2}^e = \frac{Z^2}{2}$ if $Z=\sqrt{2}$ port 2 is matched $Z_{in2}^e = 1$

$$V(x) = V^+ \cdot \left(e^{-j\beta \cdot x} + \Gamma \cdot e^{j\beta \cdot x}\right)$$

x=0 at port 1

x=-λ/4 at port 2

$$V_2^e = V(-\lambda/4) = jV^+ \cdot (1-\Gamma) = V_0$$

$$Z_{in2}^e = 1$$

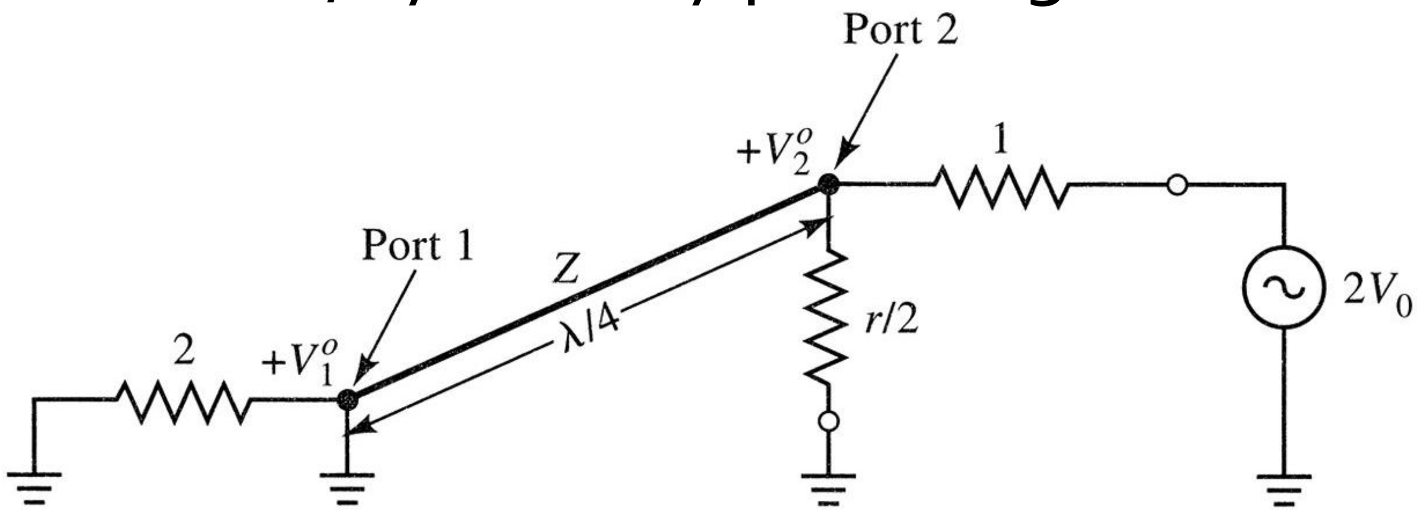
$$V_1^e = V(0) = V^+ \cdot (1+\Gamma) = jV_0 \cdot \frac{\Gamma+1}{\Gamma-1}$$

Γ: reflection coefficient seen at port 1 looking toward the resistor of normalized value 2 from the transformer $Z=\sqrt{2}$

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}} \qquad V_1^e = -jV_0\sqrt{2}$$

The Wilkinson power divider

- odd mode, symmetry plane is grounded



looking from port 2 the $\lambda/4$ line is short-circuited, impedance seen from port 2 is ∞

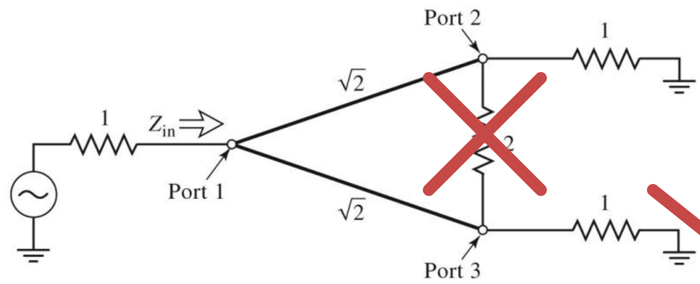
$$Z_{in2}^o = r/2 \qquad \text{if} \quad r = 2 \quad \text{port 2 is matched}$$

$$Z_{in2}^o = 1 \rightarrow V_2^o = V_0$$

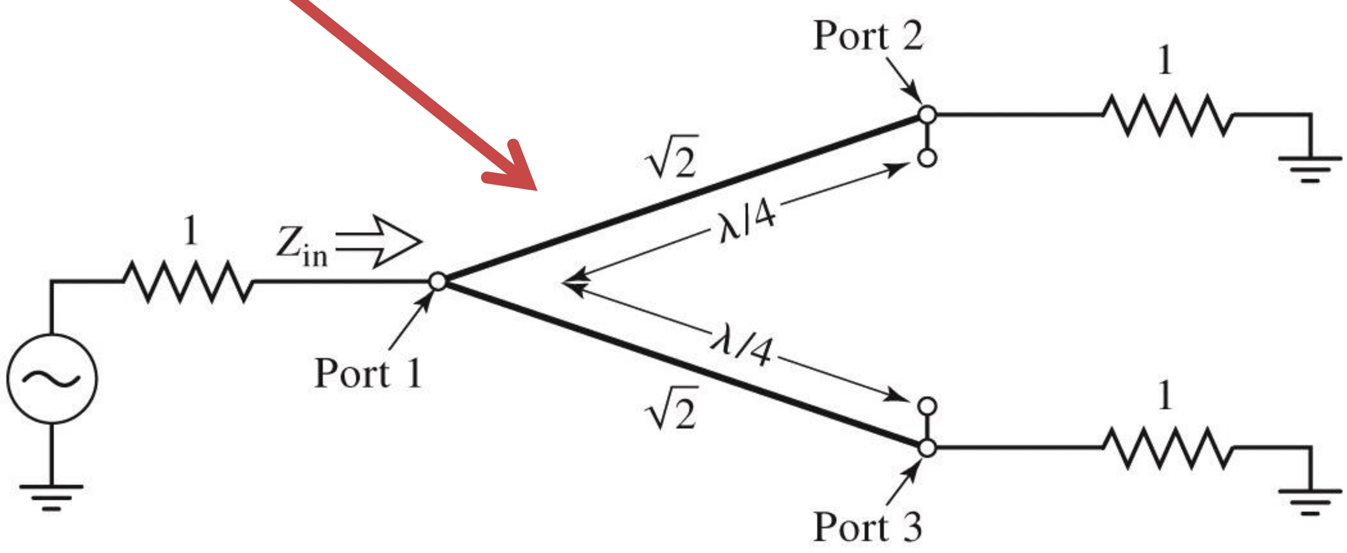
$V_1^o = 0$ in the odd mode all the power is dissipated in the *r*/2 resistor

The Wilkinson power divider

- input impedance in port 1



two $\lambda/4$ transformers with load 1 in parallel



$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1$$

The Wilkinson power divider

- S parameters

$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1 \qquad S_{11} = 0$$

$$Z_{in2}^{e} = 1 \quad Z_{in2}^{o} = 1$$ and $$Z_{in3}^{e} = 1 \quad Z_{in3}^{o} = 1 \qquad S_{22} = S_{33} = 0$$

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -\frac{j}{\sqrt{2}}$$

and $$S_{13} = S_{31} = -\frac{j}{\sqrt{2}}$$

$$S_{23} = S_{32} = 0$$ due to short or open at bisection, both eliminate transfer between the ports + reciprocal circuit

The Wilkinson power divider

- at design frequency (length of the transformer equal to $\lambda_o/4$) we have **isolation** between the two output ports

$$[S]=\begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & 0 & 0 \\ -\frac{j}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

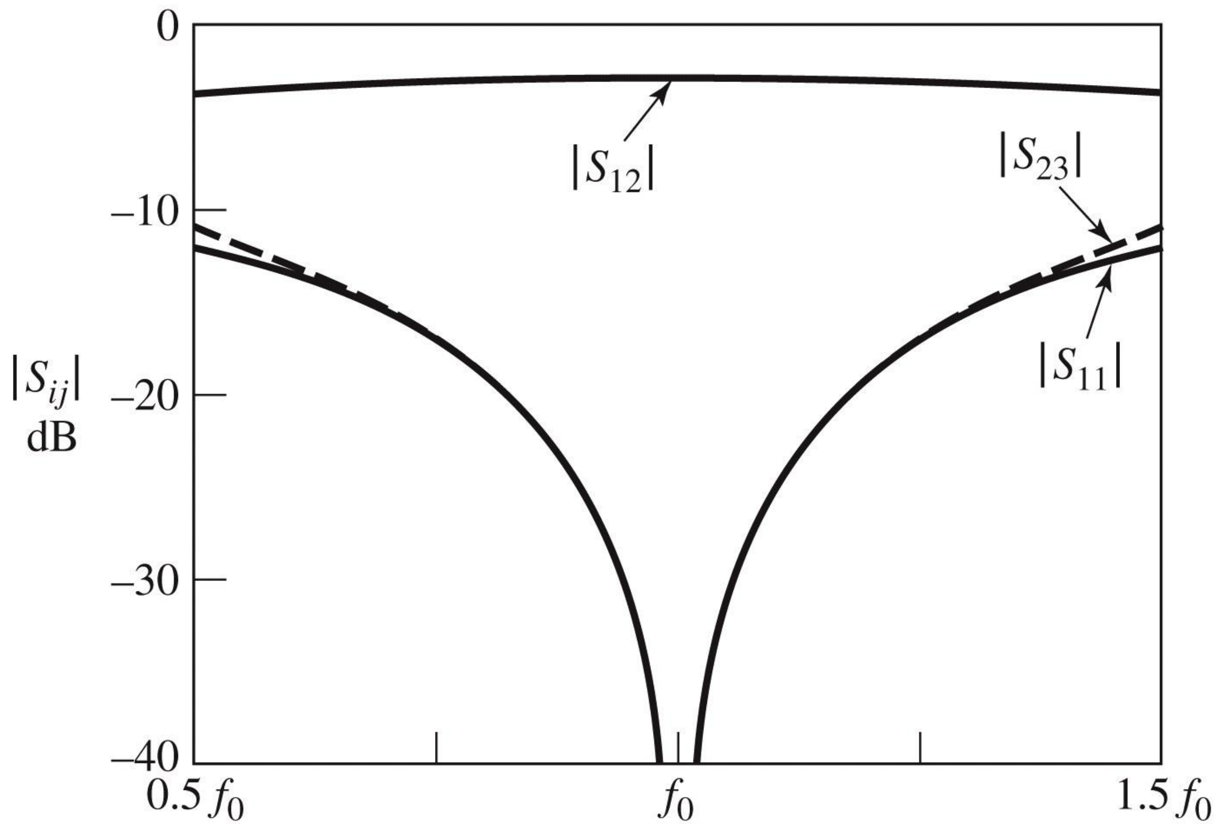


Figure 7.12

The Wilkinson power divider

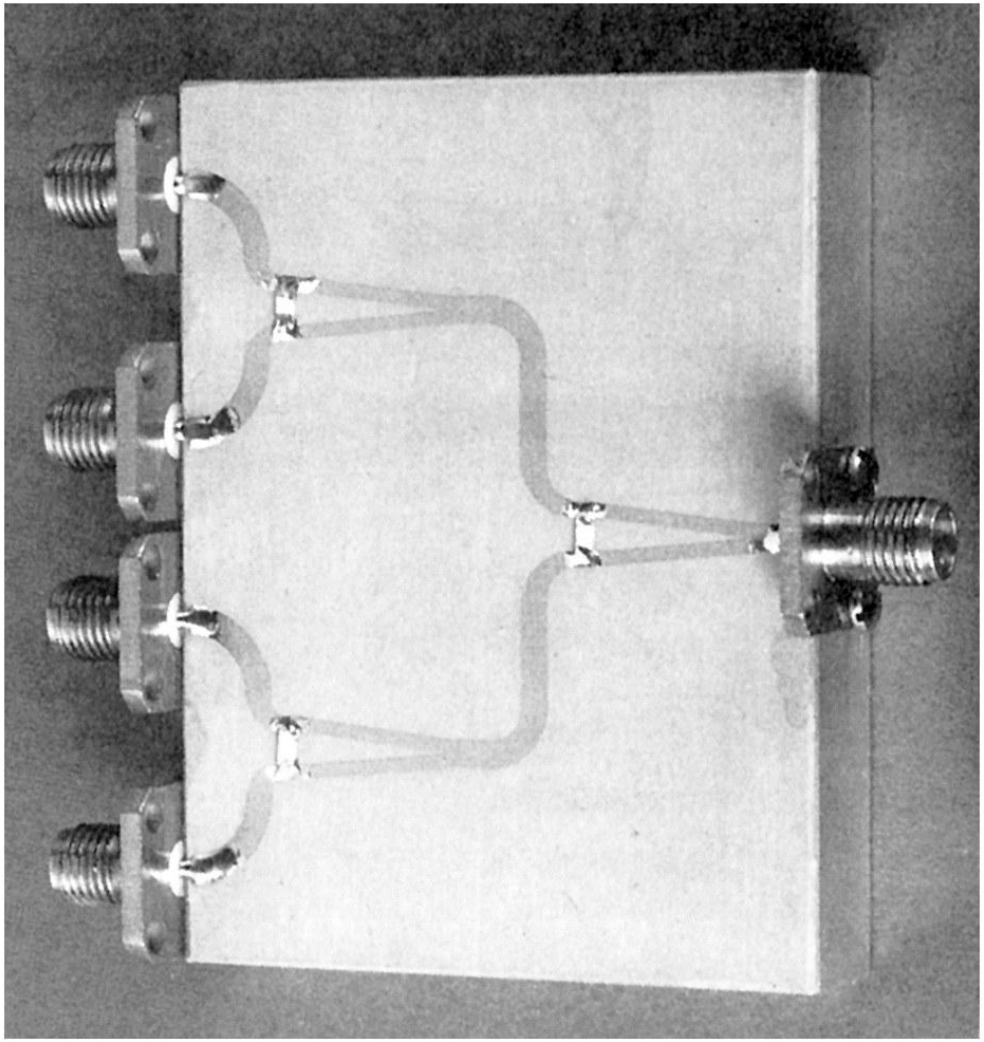
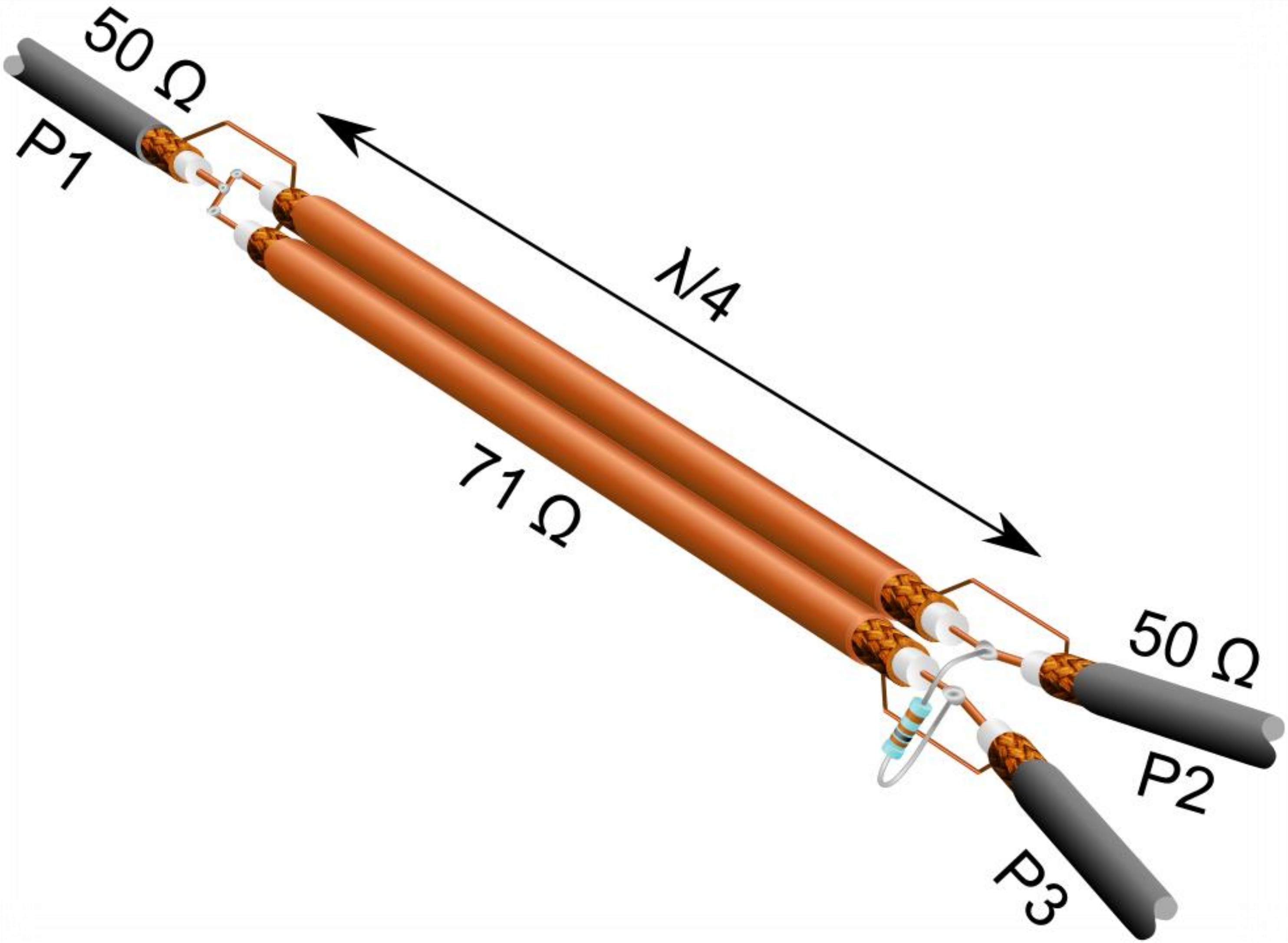


Figure 7.15
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

- 3 X Wilkinson = 4-way power divider

The Wilkinson power divider



Directional couplers

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Directional Coupler

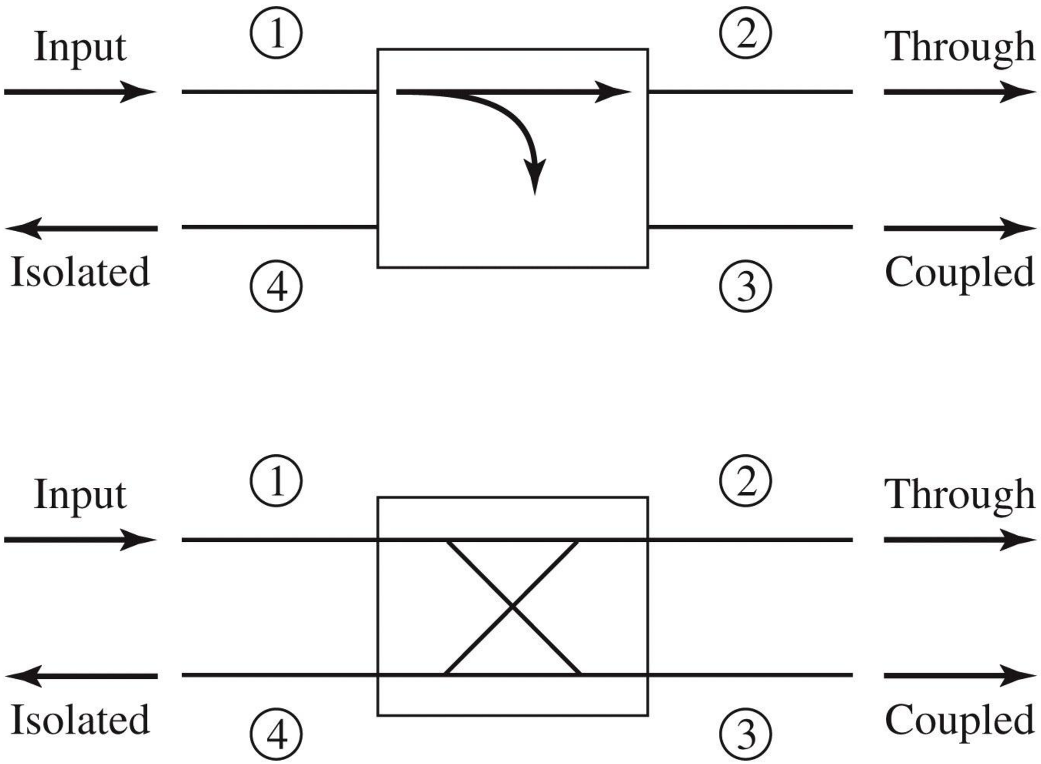


Figure 7.4

$$\left|S_{12}\right|^2 = \alpha^2 = 1 - \beta^2$$

$$\left|S_{13}\right|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{\left|S_{14}\right|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log\left|S_{14}\right|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \left[dB\right]$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Hybrid Couplers

Hybrid Couplers are directional couplers with 3 dB coupling factor

$$\alpha = \beta = 1/\sqrt{2}$$

The cuadrature (90°) hybrid

$$\left(\theta = \phi = \pi/2\right)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

The 180° ring hybrid (rat-race)

$$\left(\theta = 0\,,\phi = \pi\right)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

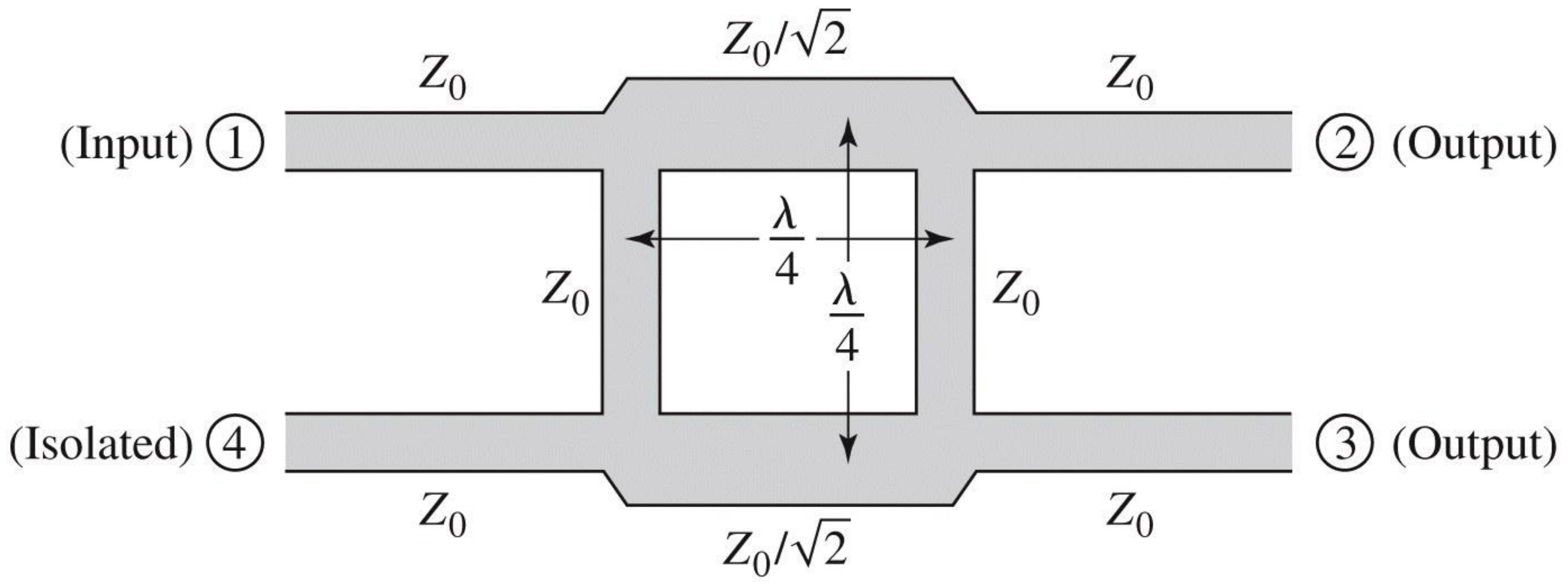
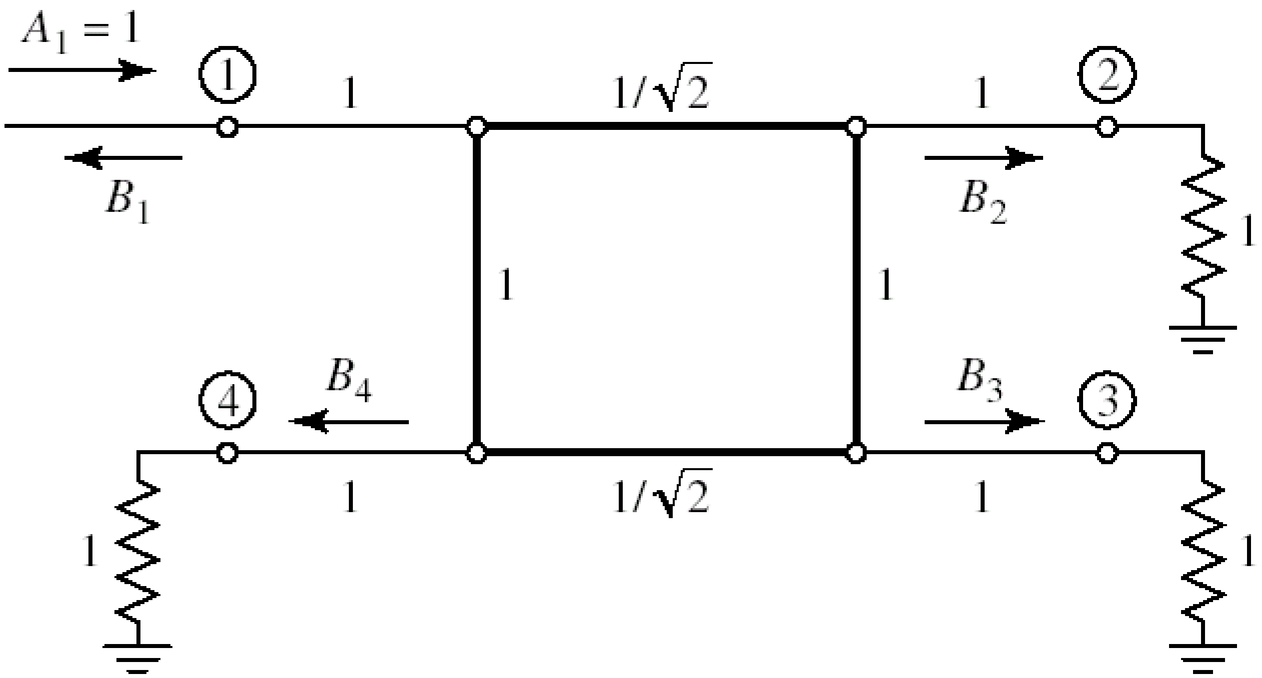


Figure 7.21

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Even/Odd Mode Analysis



Even/Odd Mode Analysis

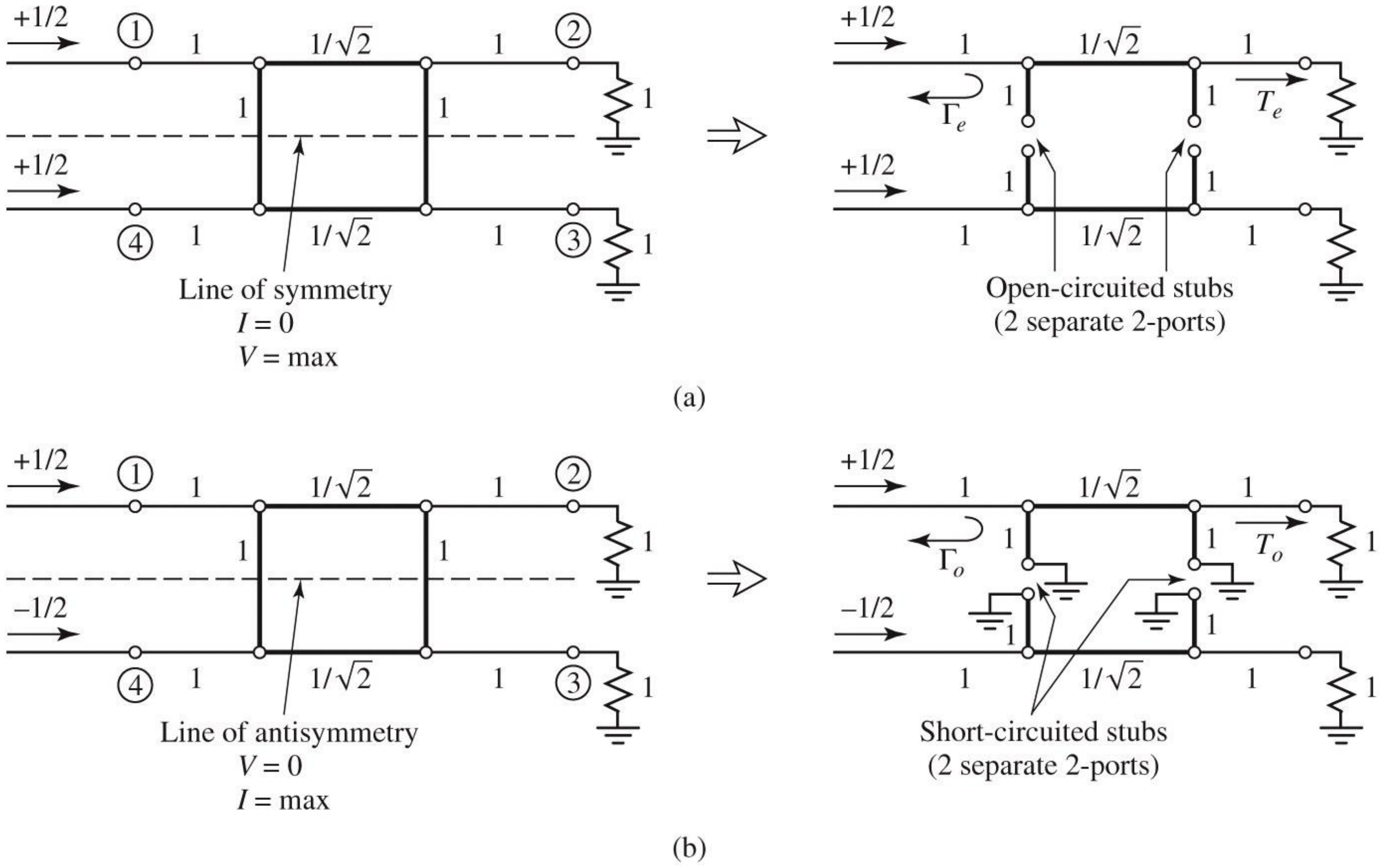
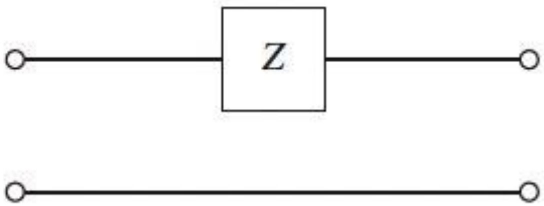
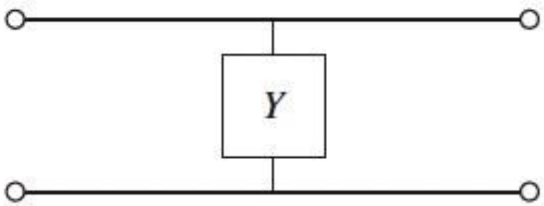
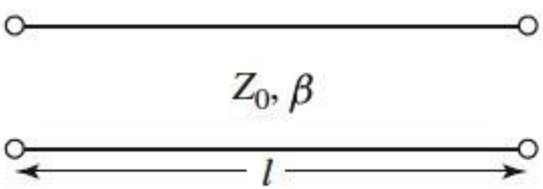


Figure 7.23

$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

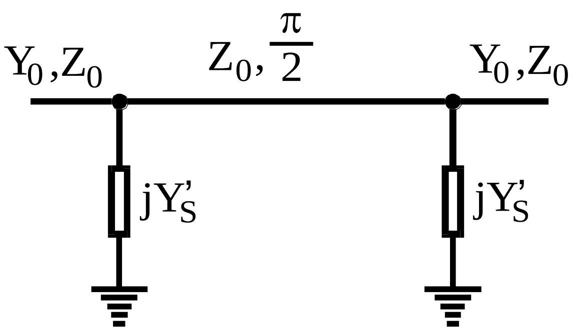
Library of ABCD matrices

TABLE 4.1 ***ABCD* Parameters of Some Useful Two-Port Circuits**

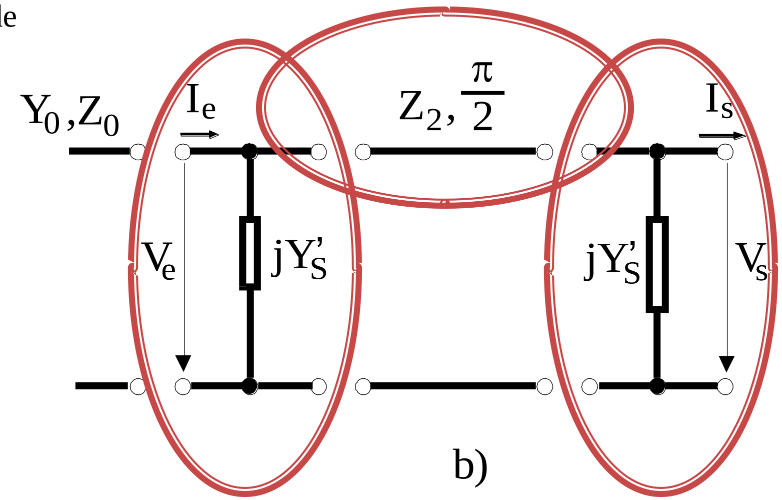
Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta\ell$ $C = jY_0 \sin \beta\ell$	$B = jZ_0 \sin \beta\ell$ $D = \cos \beta\ell$

S parameters (from ABCD)

$$Y'_S = \begin{cases} Y_1 & \text{even mode} \\ -Y_1 & \text{odd mode} \end{cases}$$



a)



b)

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & jZ_2 \\ jY_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} -Y'_S Z_2 & jZ_2 \\ -jY'^2_S Z_2 + jY_2 & -Y'_S Z_2 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$S_{11} = \frac{j\frac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S + jY_2\right)}$$

$$S_{12} = \frac{2\left[\left(-Y'_S Z_2\right)^2 - jZ_2\left(-jY'^2_S Z_2 + jY_2\right)\right]}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$\Gamma = S_{11} = \frac{j\left(z_2 - y_2 + y'^2_S z_2\right)}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{22}$$

$$S_{21} = \frac{2}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$S_{22} = \frac{j\frac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$T = S_{21} = \frac{2}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{12}$$

Relation between two port S parameters and ABCD parameters

$$A=\sqrt{\frac{Z_{01}}{Z_{02}}}\frac{(1+S_{11}-S_{22}-\Delta S)}{2S_{21}}$$

$$B=\sqrt{Z_{01}Z_{02}}\frac{(1+S_{11}+S_{22}+\Delta S)}{2S_{21}}$$

$$C=\frac{1}{\sqrt{Z_{01}Z_{02}}}\frac{1-S_{11}-S_{22}+\Delta S}{2S_{21}}$$

$$D=\sqrt{\frac{Z_{02}}{Z_{01}}}\frac{1-S_{11}+S_{22}-\Delta S}{2S_{21}}$$

$$\Delta S=S_{11}S_{22}-S_{12}S_{21}$$

$$S_{11}=\frac{AZ_{02}+B-CZ_{01}Z_{02}-DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{12}=\frac{2(AD-BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{21}=\frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{22}=\frac{-AZ_{02}+B-CZ_{01}Z_{02}+DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

Matching and coupling factor

$$\Gamma_e = \frac{j\cdot\left(z_2 - y_2 + y_1^2 z_2\right)}{-2y_1z_2 + j\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$\Gamma_o = \frac{j\cdot\left(z_2 - y_2 + y_1^2 z_2\right)}{2y_1z_2 + j\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$T_e = \frac{2}{-2y_1z_2 + j\cdot\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$T_o = \frac{2}{2y_1z_2 + j\cdot\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$b_1 = \frac{\Gamma_e + \Gamma_o}{2} = \frac{z_2^2 - \left(y_2 - y_1^2 z_2\right)^2}{\left(2y_1z_2\right)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_2 = \frac{T_e + T_o}{2} = \frac{-2j\left(z_2 + y_2 - y_1^2 z_2\right)}{\left(2y_1z_2\right)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_3 = \frac{T_e - T_o}{2} = \frac{-4y_1z_2}{\left(2y_1z_2\right)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_4 = \frac{\Gamma_e - \Gamma_o}{2} = \frac{-2jy_1z_2\left(z_2 - y_2 + y_1^2 z_2\right)}{\left(2y_1z_2\right)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_1 = 0 \Rightarrow z_2 - y_2 + y_1^2 z_2 = 0 \Rightarrow z_2^2 = \frac{1}{1 + y_1^2}$$

$$\boxed{y_2^2 = 1 + y_1^2}$$

$$\boxed{C = 10\log\frac{P_1}{P_3} = -20\log\left|b_3\right|,dB}$$

$$b_1 = 0 \quad b_4 = 0 \quad b_3 = -y_1z_2 \quad b_2 = -jz_2$$

$$b_3 = -\frac{\sqrt{y_2^2 - 1}}{y_2},\ b_2 = -\frac{j}{y_2}$$

$$\boxed{\beta = \frac{\sqrt{y_2^2 - 1}}{y_2}}$$

$$b_3 = -C$$

$$b_2 = -j\sqrt{1 - C^2}$$

$$[S] = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & -C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & -C \\ -C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & -C & -j\sqrt{1-C^2} & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

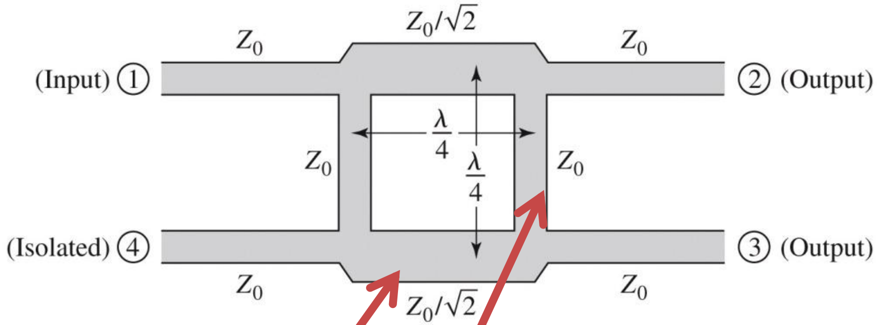


Figure 7.21

$$\boxed{y_2^2 = 1 + y_1^2}$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$\boxed{C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}}$$

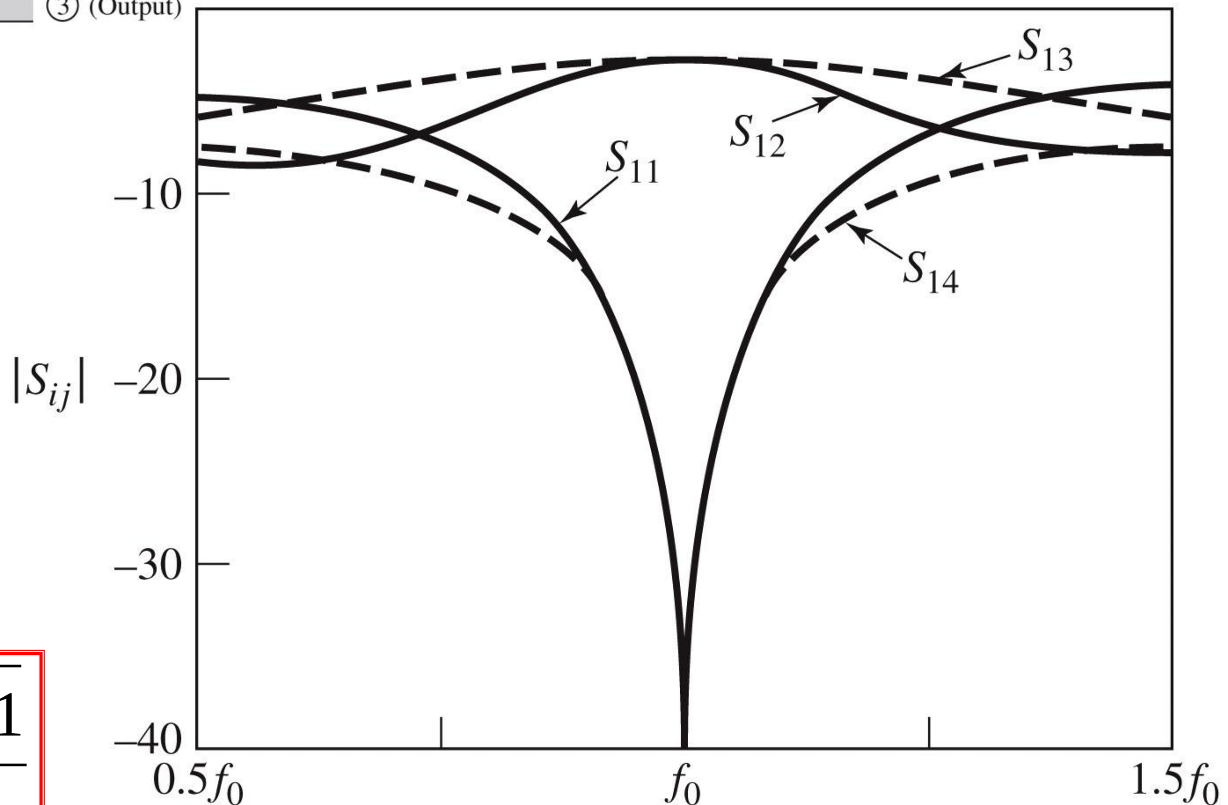


Figure 7.25

Example

Design a cuadrature (90°) hybrid working on 50 Ω, and plot the S parameters between

$0.5 f_0$ and $1.5 f_0$, where f_0

is the frequency at which the length of the branches is $\lambda/4$

.

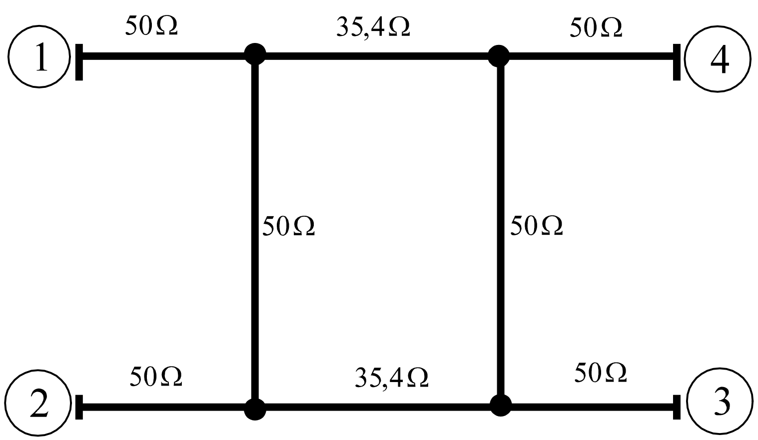
Solution

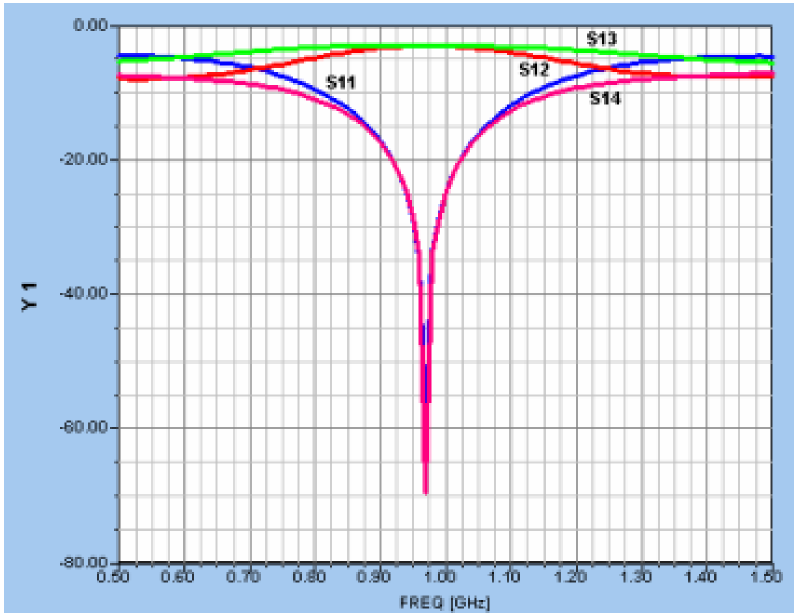
A cuadrature (90°) hybrid has C = 3dB, then $\beta = 1/\sqrt{2}$

$$y_2 = \sqrt{2} \quad \text{and} \quad y_1 = 1$$

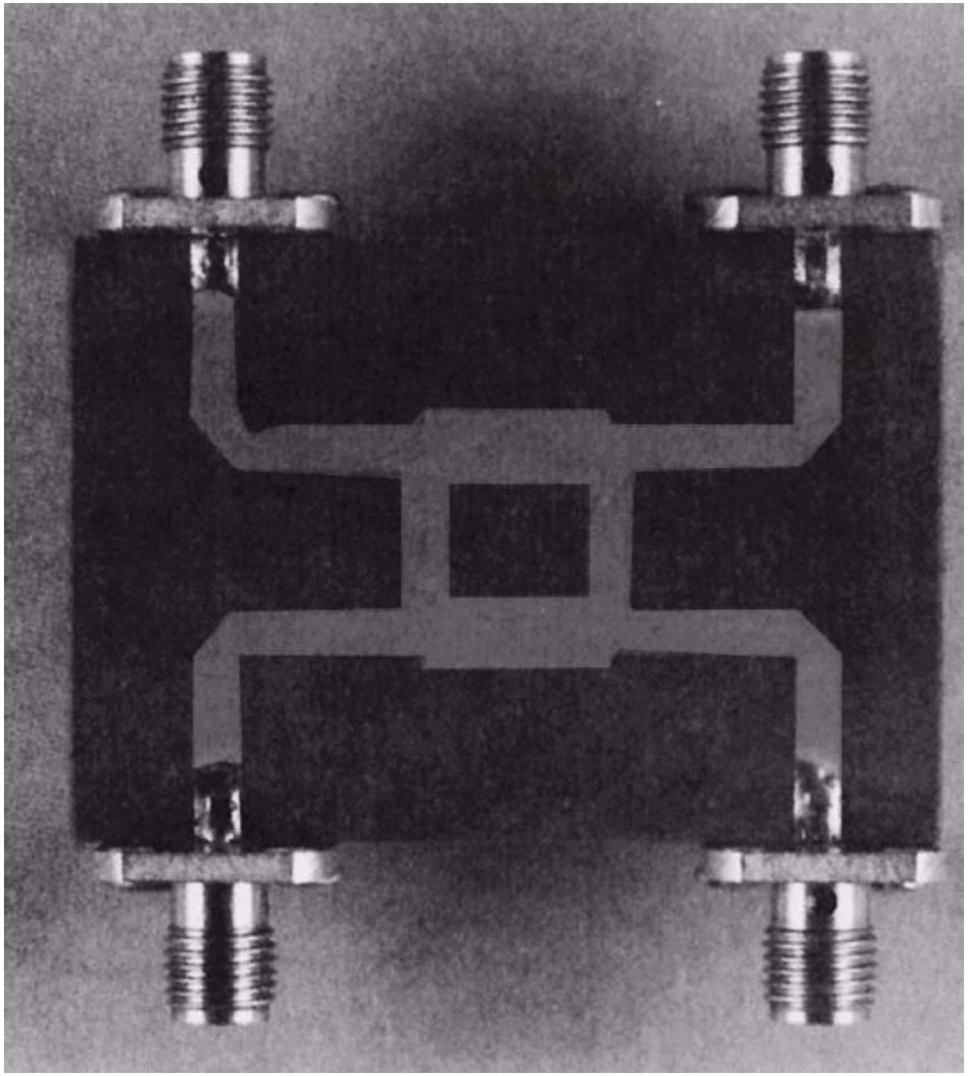
$Z_0 = 50\Omega$ the characteristic impedances will be:

$$Z_1 = Z_0 = 50\Omega \qquad Z_2 = \frac{Z_0}{\sqrt{2}} = 35.4\Omega$$





The cuadrature (90°) hybrid



The cuadrature (90°) hybrid

- eight-way microstrip power divider with six quadrature hybrids in a Bailey configuration

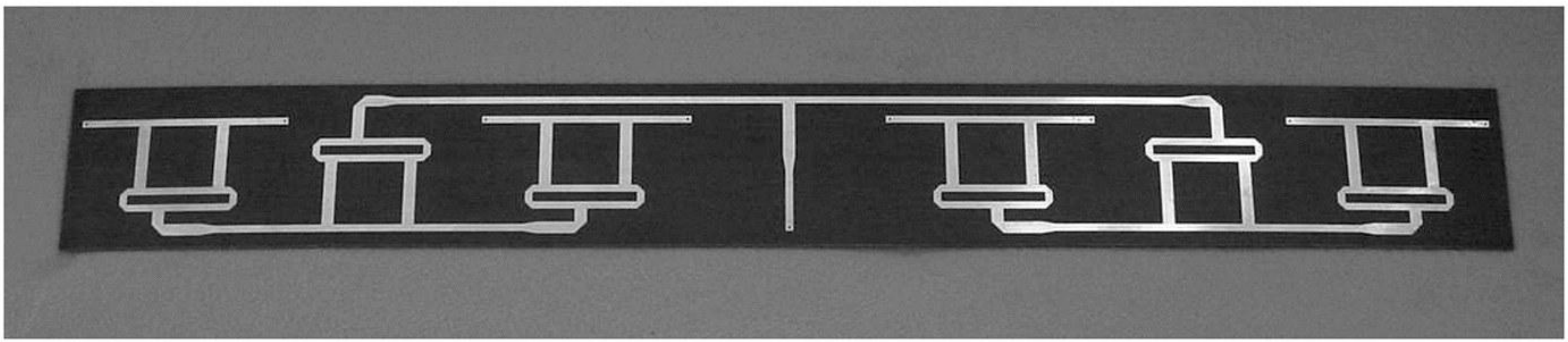
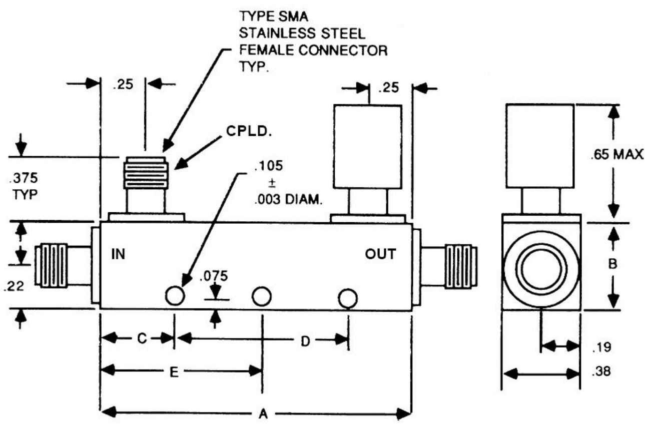


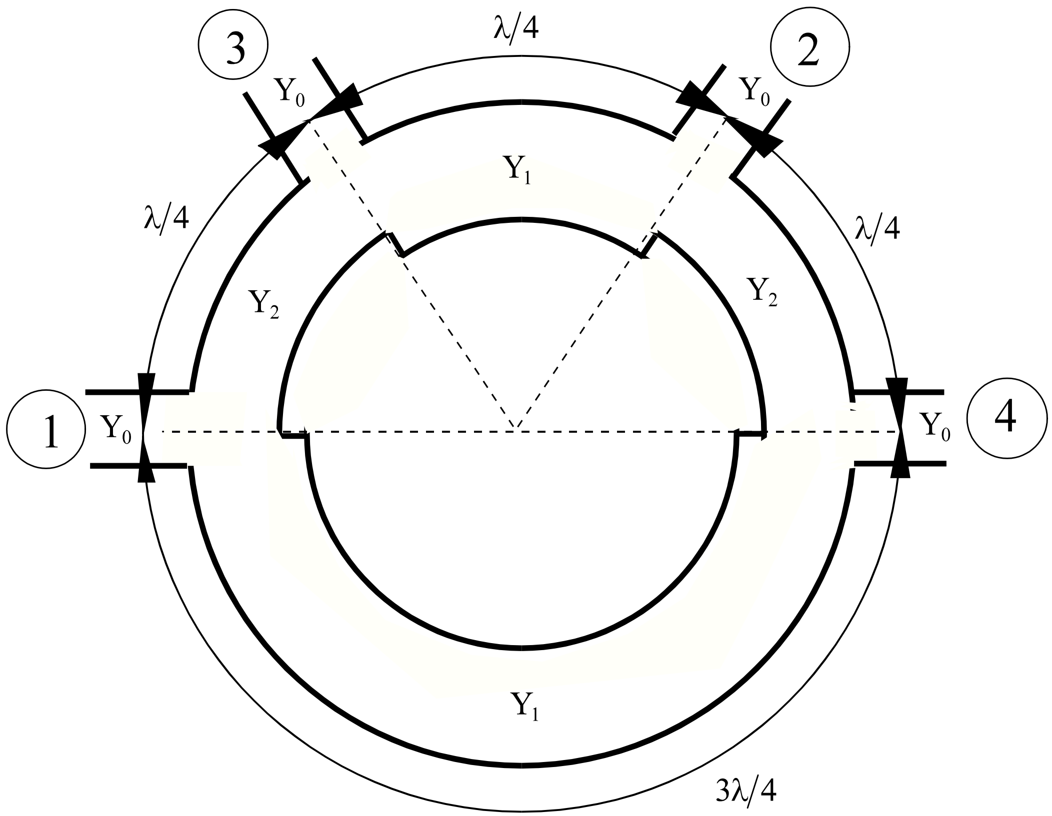
Figure 7.24
Courtesy of ProSensing, Inc., Amherst, Mass.

Datasheet



Model No.	Frequency Range (Ghz)	Coupling † (dB)	Freq. Sens. (dB)	Insertion Loss (dB)		Directivity (dB min.)	VSWR max.	
				Excl. Cpld Pwr	True		Primary Line	Secondary Line
MDC6223-6	0.5-1.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6223-10	0.5-1.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6223-20	0.5-1.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6223-30	0.5-1.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-6	1.0-2.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6224-10	1.0-2.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6224-20	1.0-2.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-30	1.0-2.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6225-6	2.0-4.0	6 ±1.00	±0.60	0.20	1.80	22	1.15	1.15
MDC6225-10	2.0-4.0	10 ±1.25	±0.75	0.20	0.80	22	1.15	1.15
MDC6225-20	2.0-4.0	20 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6225-30	2.0-4.0	30 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6266-6	2.6-5.2	6 ±1.00	±0.60	0.20	1.80	20	1.25	1.25
MDC6266-10	2.6-5.2	10 ±1.25	±0.75	0.20	0.80	20	1.25	1.25
MDC6266-20	2.6-5.2	20 ±1.25	±0.75	0.20	0.25	20	1.25	1.25
MDC6266-30	2.6-5.2	30 ±1.25	±0.75	0.20	0.20	20	1.25	1.25
MDC6226-6	4.0-8.0	6 ±1.00	±0.60	0.25	1.90	20	1.25	1.25
MDC6226-10	4.0-8.0	10 ±1.25	±0.75	0.25	0.90	20	1.25	1.25
MDC6226-20	4.0-8.0	20 ±1.25	±0.75	0.25	0.30	20	1.25	1.25
MDC6226-30	4.0-8.0	30 ±1.25	±0.75	0.25	0.25	20	1.25	1.25
MDC6227-6	7.0-12.4	6 ±1.00	±0.50	0.30	2.00	17	1.30	1.30
MDC6227-10	7.0-12.4	10 ±1.00	±0.50	0.30	1.00	17	1.30	1.30
MDC6227-20	7.0-12.4	20 ±1.00	±0.50	0.30	0.35	17	1.30	1.30
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

The 180° ring hybrid (rat-race)



The 180° ring hybrid

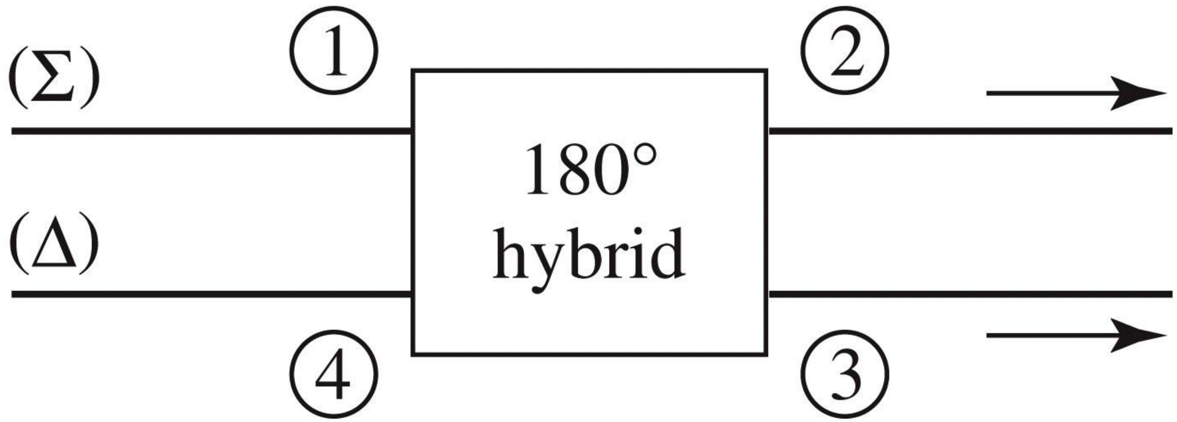
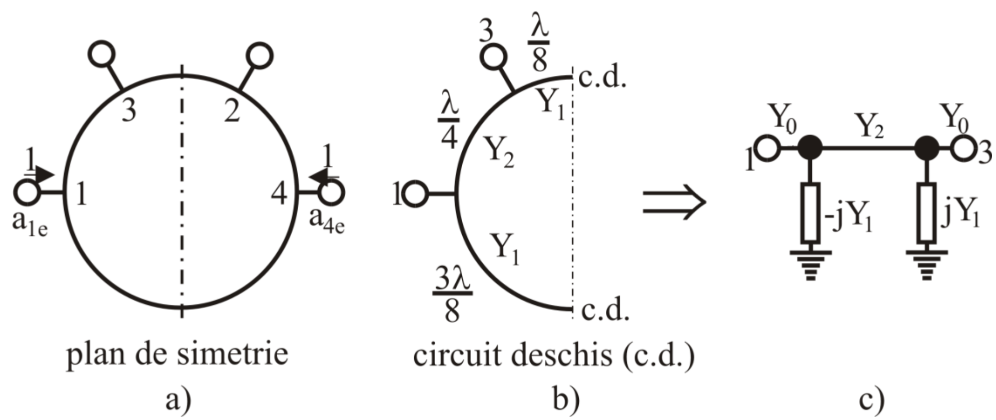


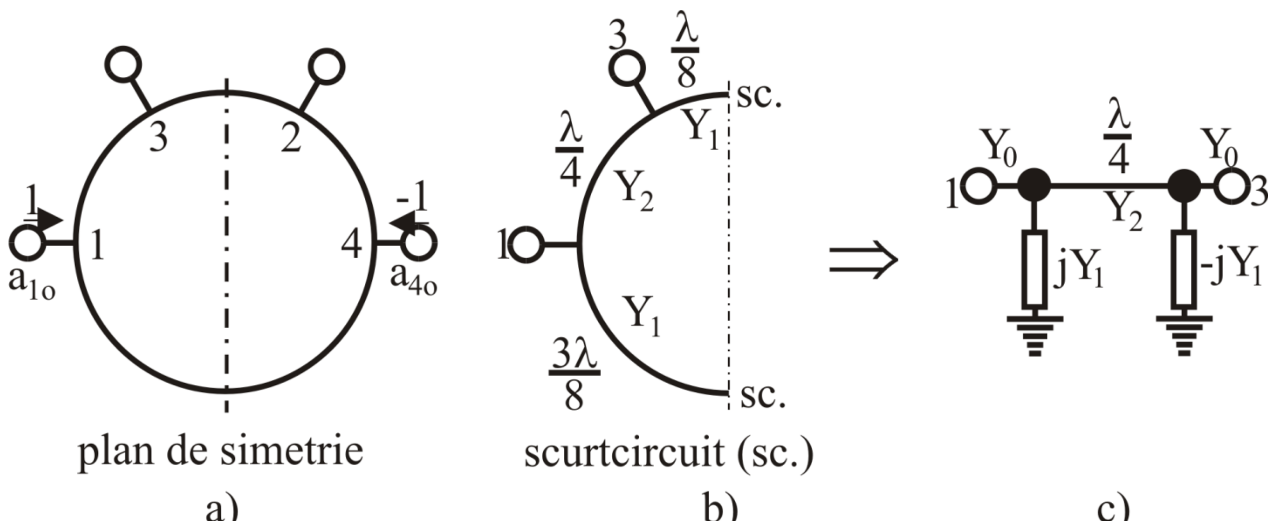
Figure 7.41

- The 180° ring hybrid can be operated in different modes:
  - a signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3
  - input applied to port 4 it will be equally split into two components with a 180° phase difference at ports 2 and 3
  - input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4 (power combiner)

Even/Odd Mode Analysis



Even Mode

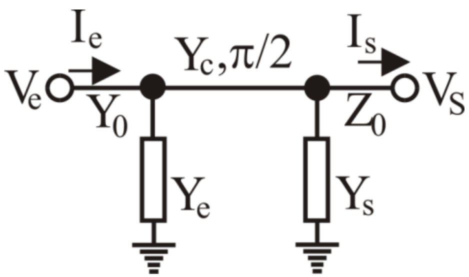


Odd Mode

Even/Odd Mode Analysis

$$S_{11} = \frac{jz_2 y_s + jz_2 - j(y_2 + y_e y_s z_2) - jy_e z_2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

$$S_{12} = \frac{2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$



$$S_{21} = \frac{2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

$$S_{22} = \frac{-jz_2 y_s + jz_2 - j(y_2 + y_e y_s z_2) + jy_e z_2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

Even mode:

$$y_e = -jy_1$$
$$y_s = jy_1$$

$$S_{11e} = \frac{z_2 - y_2 - y_1^2 z_2 + 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{12e} = S_{21e} = \frac{-2j}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{22e} = \frac{z_2 - y_2 - y_1^2 z_2 - 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

Matching condition

$$y_1^2 + y_2^2 = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & -jy_2 & jy_1 \\ 0 & 0 & -jy_1 & -jy_2 \\ -jy_2 & -jy_1 & 0 & 0 \\ jy_1 & -jy_2 & 0 & 0 \end{bmatrix}$$

Odd mode:

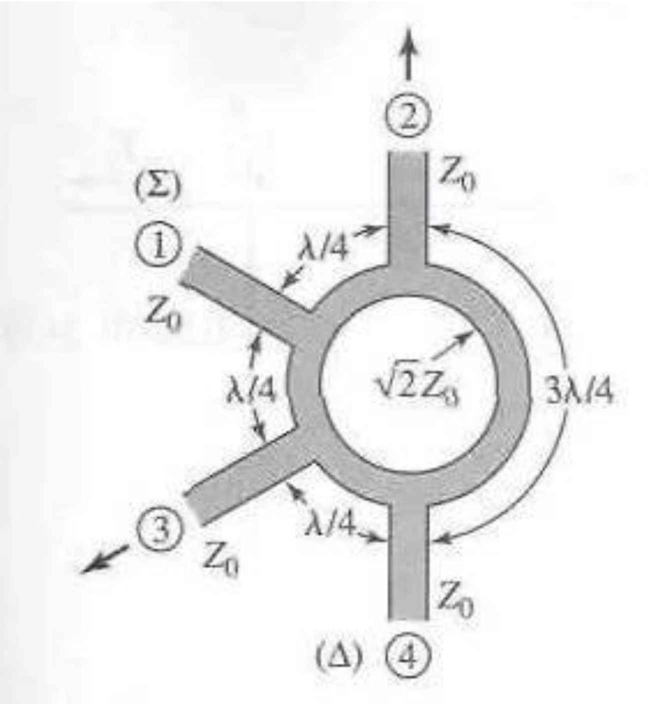
$$y_e = jy_1$$
$$y_s = -jy_1$$

$$S_{11o} = \frac{z_2 - y_2 - y_1^2 z_2 - 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{12o} = S_{21o} = \frac{-2j}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{22o} = \frac{z_2 - y_2 - y_1^2 z_2 + 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

The 180° ring hybrid



$$[S]=\begin{bmatrix} 0 & -jy_2 & -jy_1 & 0 \\ -jy_2 & 0 & 0 & jy_1 \\ -jy_1 & 0 & 0 & -jy_2 \\ 0 & jy_1 & -jy_2 & 0 \end{bmatrix} = -j\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$C(dB) = -20\log(\beta) = -20\log(y_1)$$

Example

Design a ring (180°) hybrid working on 50 Ω, and plot the S parameters between 0.5 and 1.5 of the design frequency.

$$C\,[\mathrm{dB}] = -20\log(y_1)$$

$$\sqrt{2}Z_0 = 70.7\Omega$$

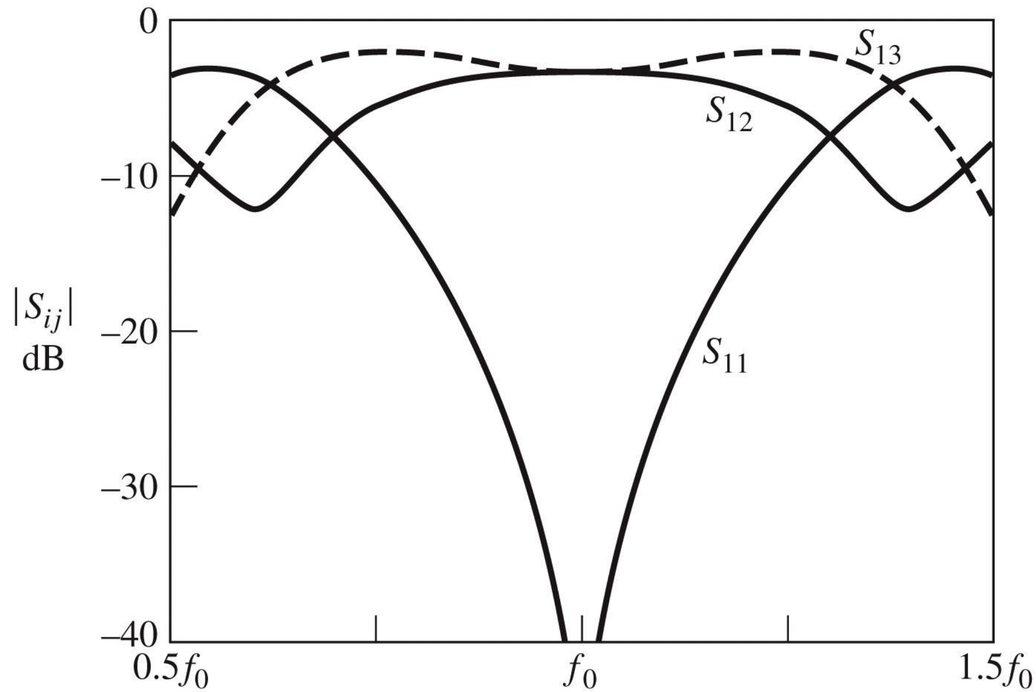
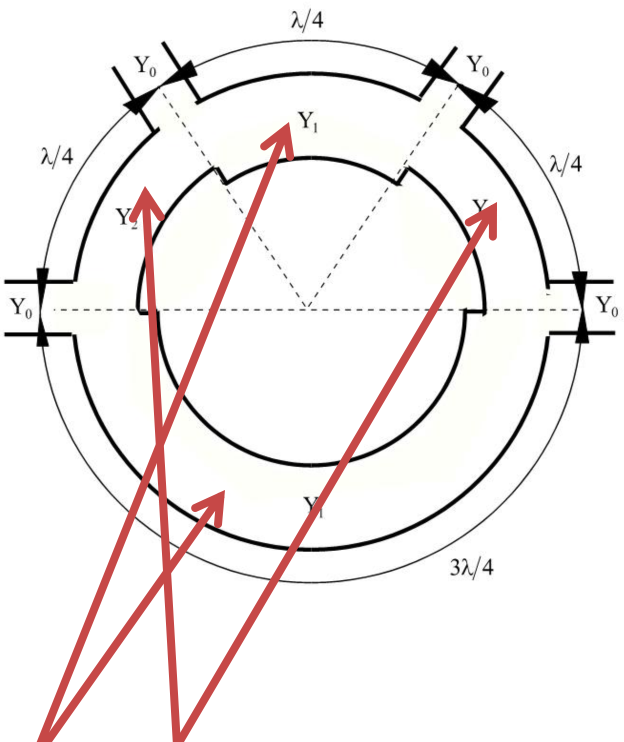


Figure 7.46

The 180° ring hybrid



$$\boxed{y_1^2 + y_2^2 = 1}$$

$$|\beta| = y_1$$

$$\boxed{C\,[\mathrm{dB}] = -20 \cdot \log_{10}(y_1)}$$

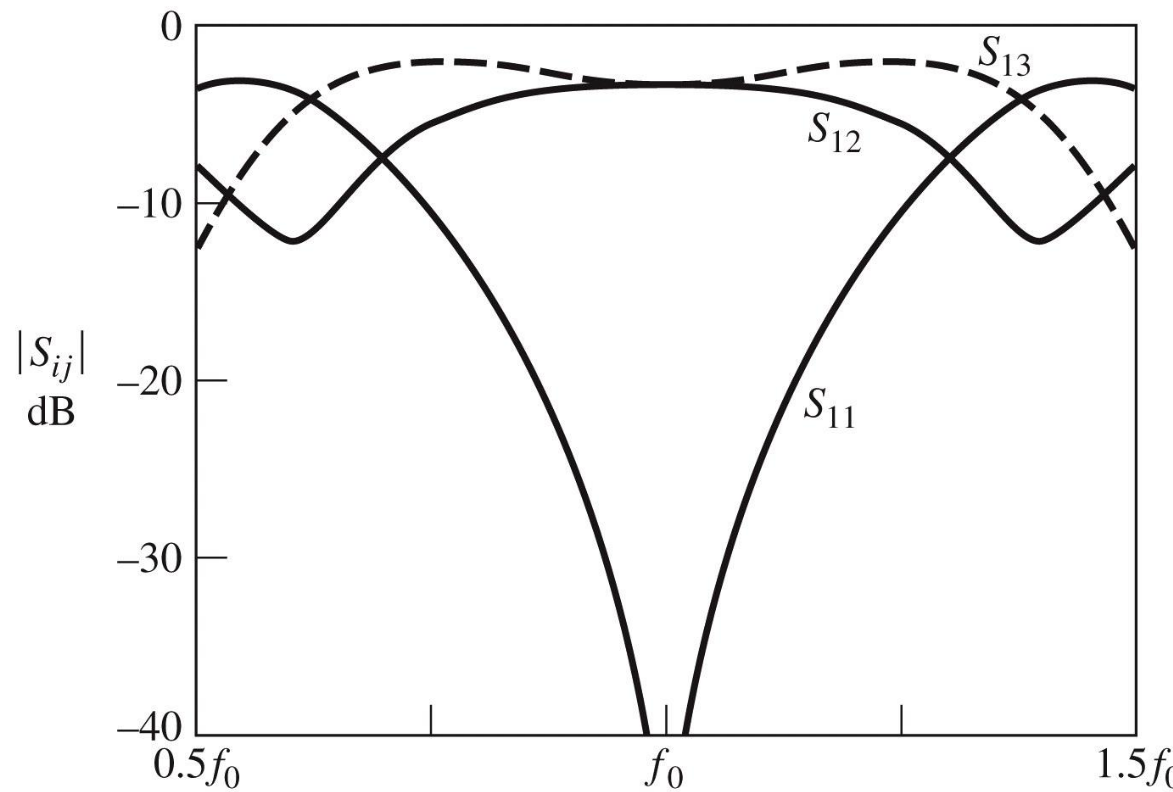


Figure 7.46

The 180° ring hybrid

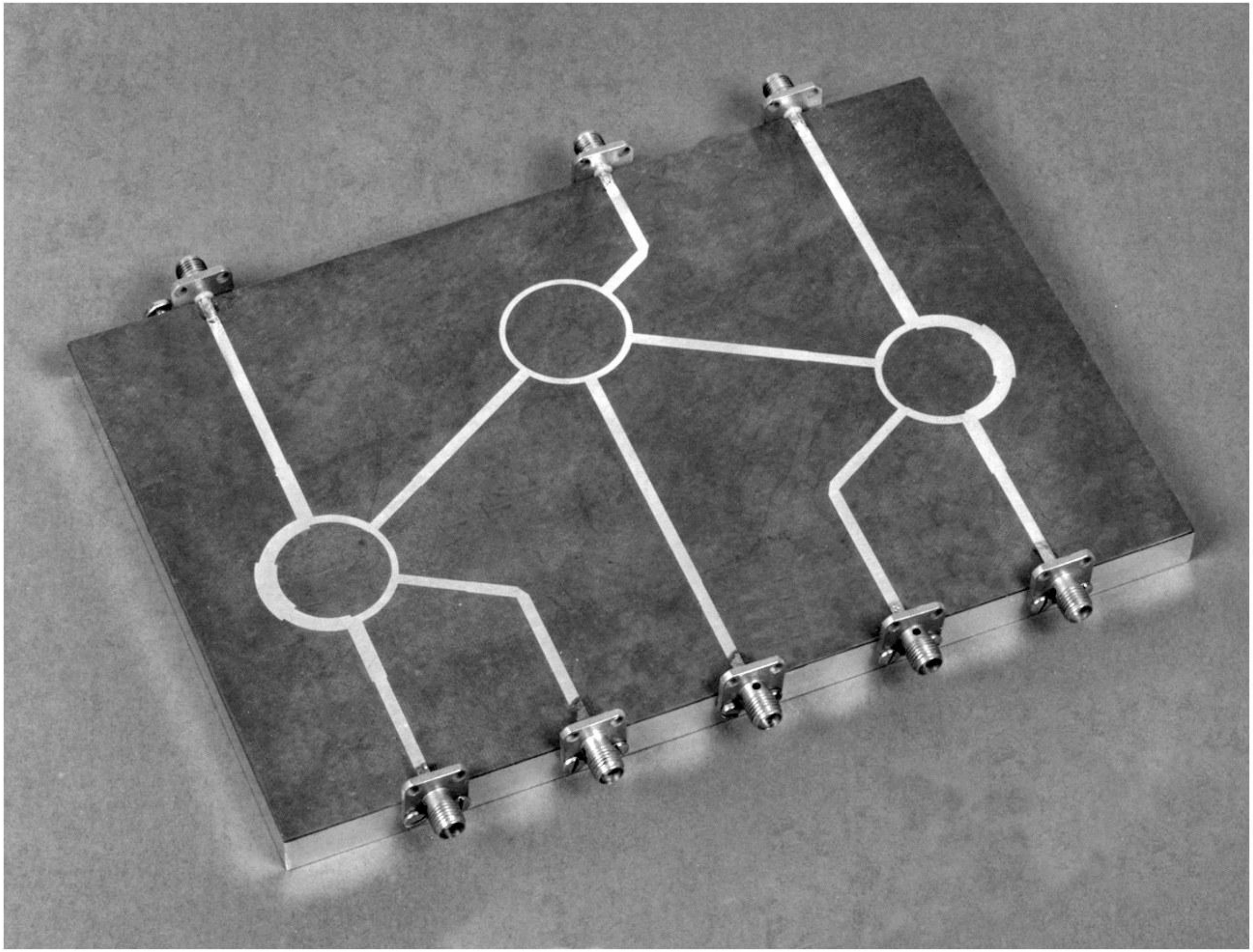
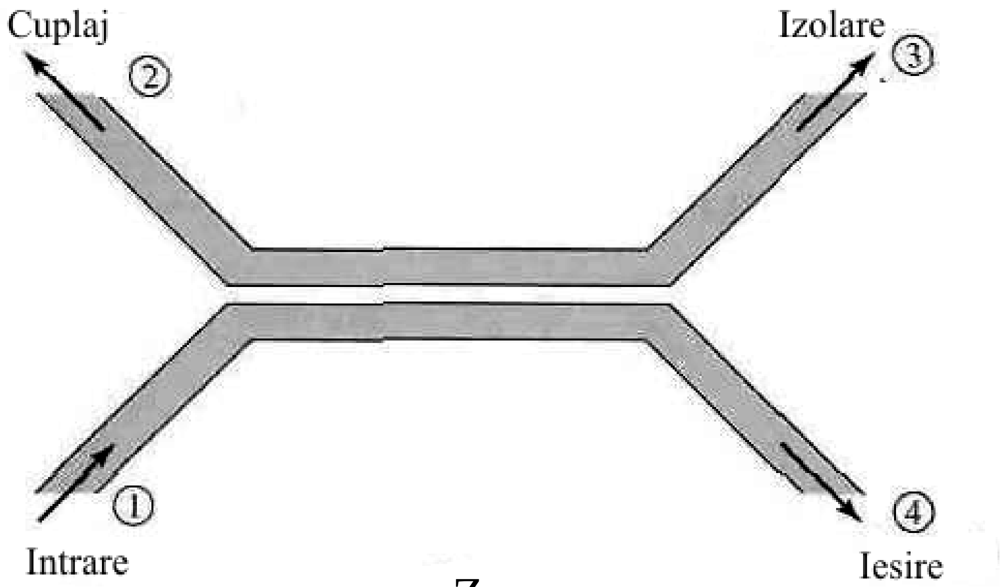
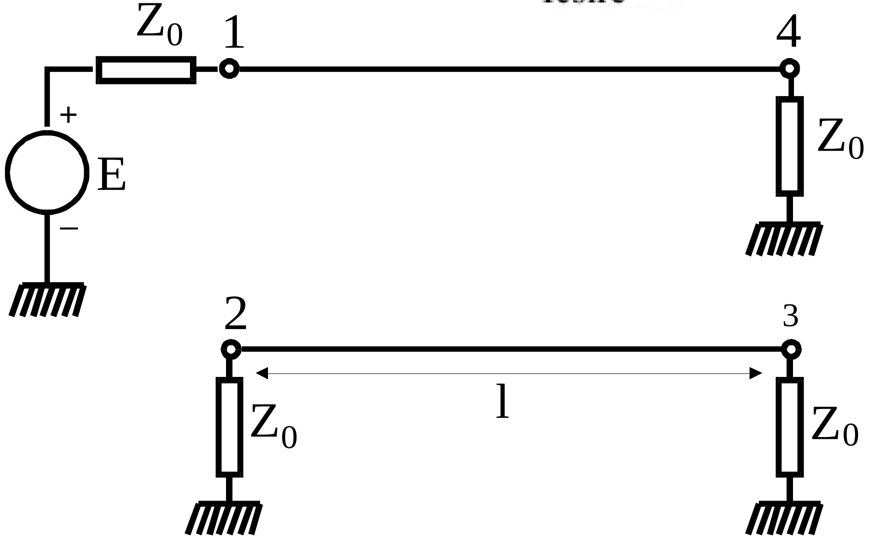


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler





Coupled Lines

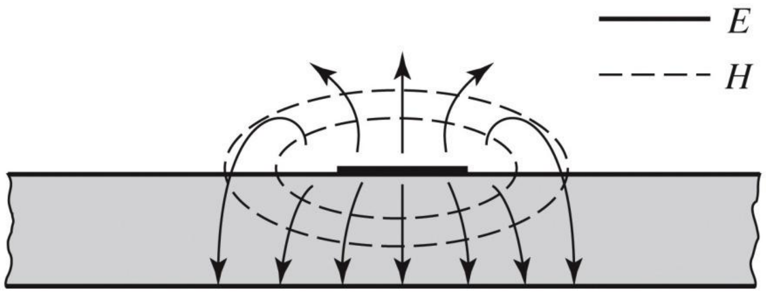
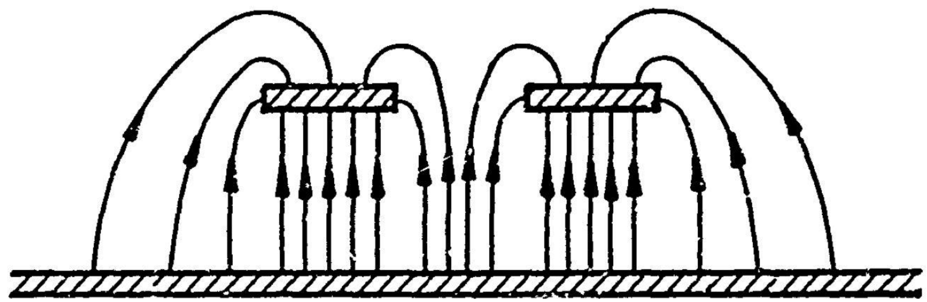
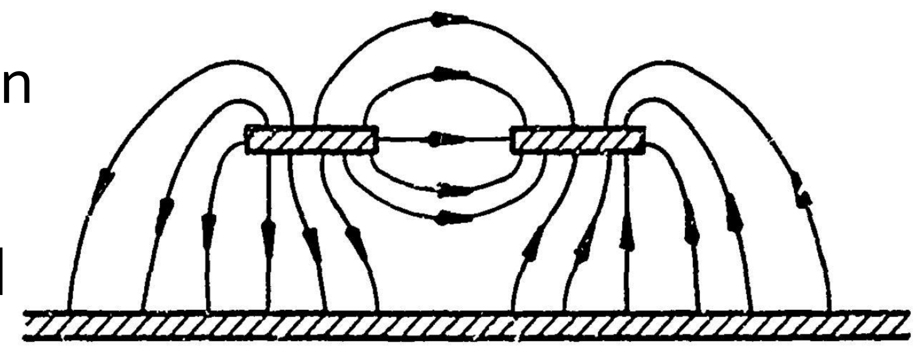


Figure 3.25b



b) EVEN MODE ELECTRIC FIELD PATTERN (SCHEMATIC)



c) ODD MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

- Even mode - characterizes the common mode signal on the two lines
- Odd mode - characterizes the differential mode signal between the two lines
- Each of the two modes is characterized by **different** characteristic impedances

Coupled Lines

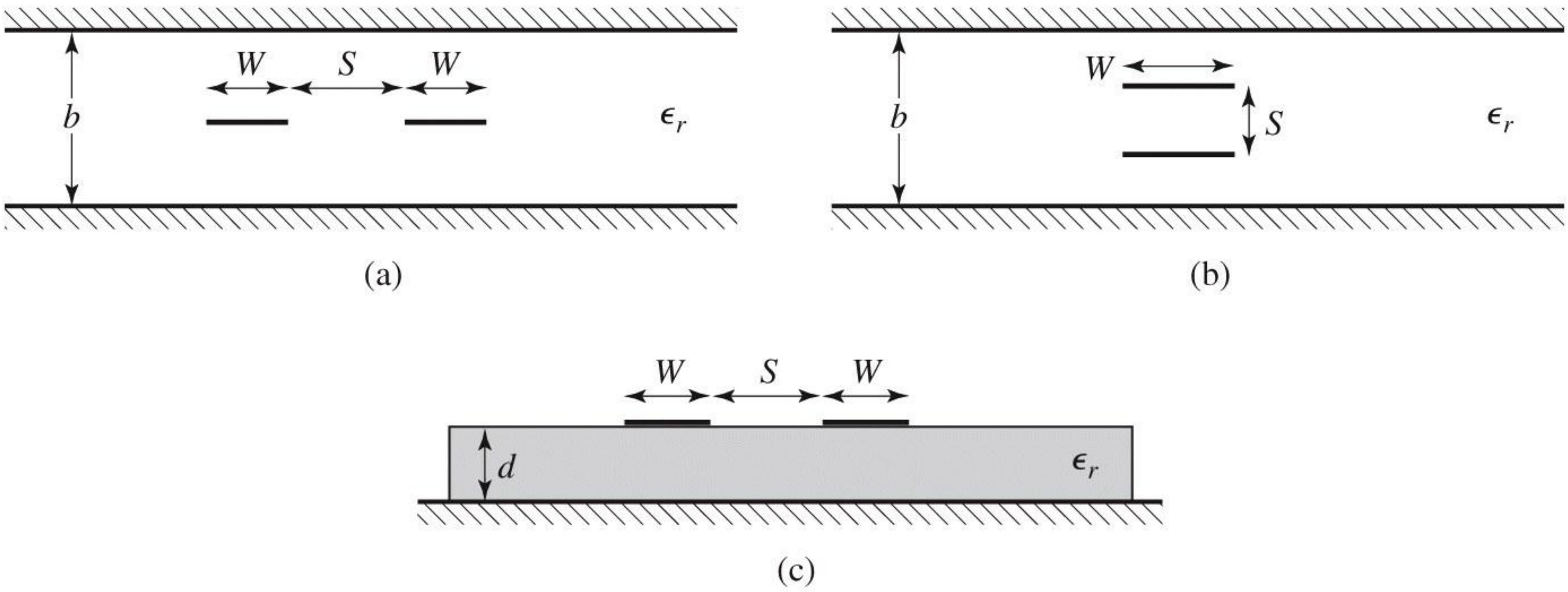


Figure 7.26

Coupled Lines

Figure 7.27

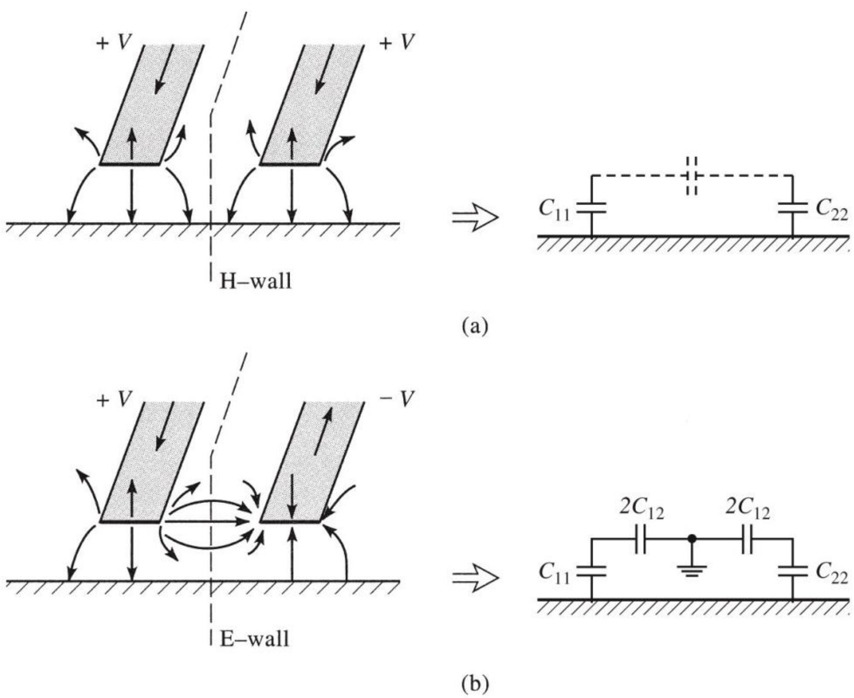
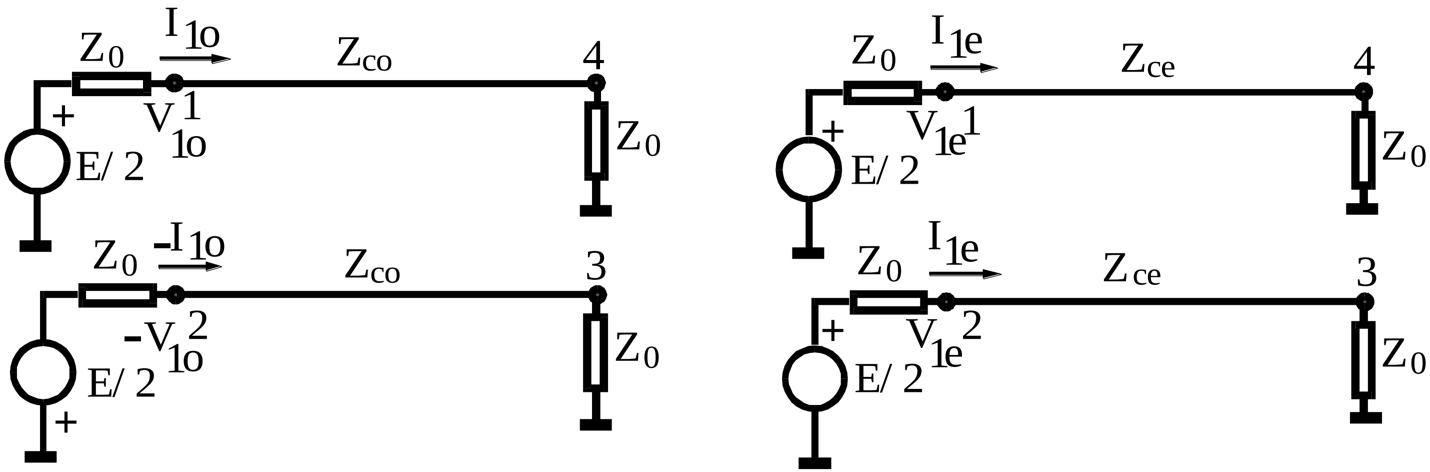


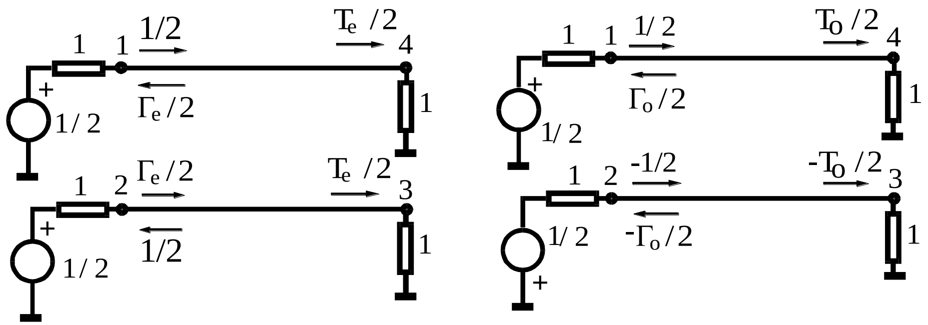
Figure 7.28

Matching in Coupled Line Coupler



$$\begin{cases} Z_{ce}Z_{co} = Z_0^2 \\ \theta_e = \theta_o \end{cases}$$

Directivity and Coupling factor



modul par

modul impar

$$a_1 = a_{1e} + a_{1o} = 1\,,\; a_2 = a_3 = a_4 = 0$$

$$b_1 = \frac{1}{2}\left(\Gamma_e + \Gamma_o\right) = 0 \Leftrightarrow$$

$$b_2 = \frac{1}{2}\left(\Gamma_e - \Gamma_o\right) = \frac{jC\sin(\theta)}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$b_3 = \frac{1}{2}\left(T_e - T_o\right) = 0$$

$$b_4 = \frac{1}{2}\left(T_e + T_o\right) = \frac{\sqrt{1-C^2}}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$\boxed{C = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}}$$

$$\boxed{\theta = \pi/2}$$

$$\boxed{[S] = \begin{bmatrix} 0 & C & 0 & -j\sqrt{1-C^2} \\ C & 0 & -j\sqrt{1-C^2} & 0 \\ 0 & -j\sqrt{1-C^2} & 0 & C \\ -j\sqrt{1-C^2} & 0 & C & 0 \end{bmatrix}}$$

Coupled Line Coupler

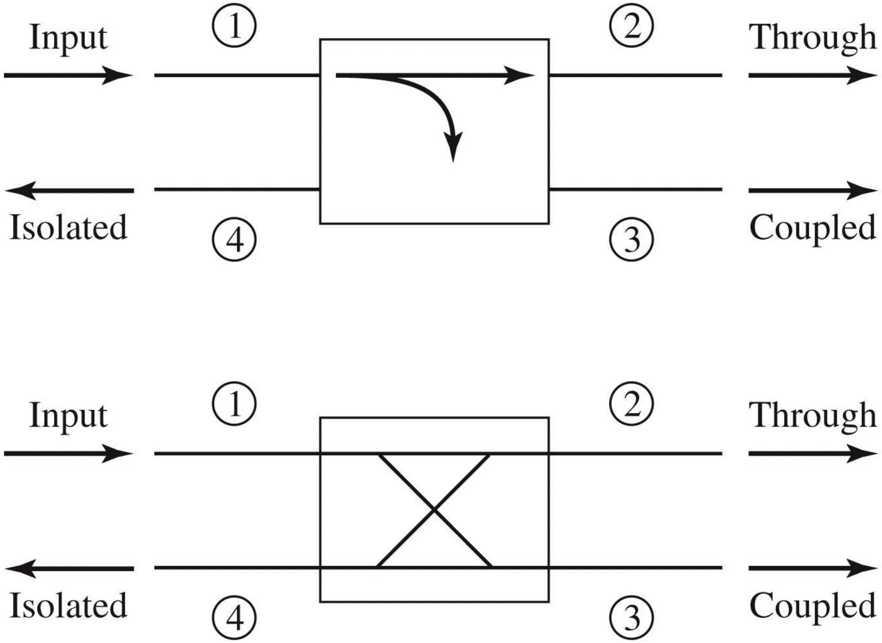
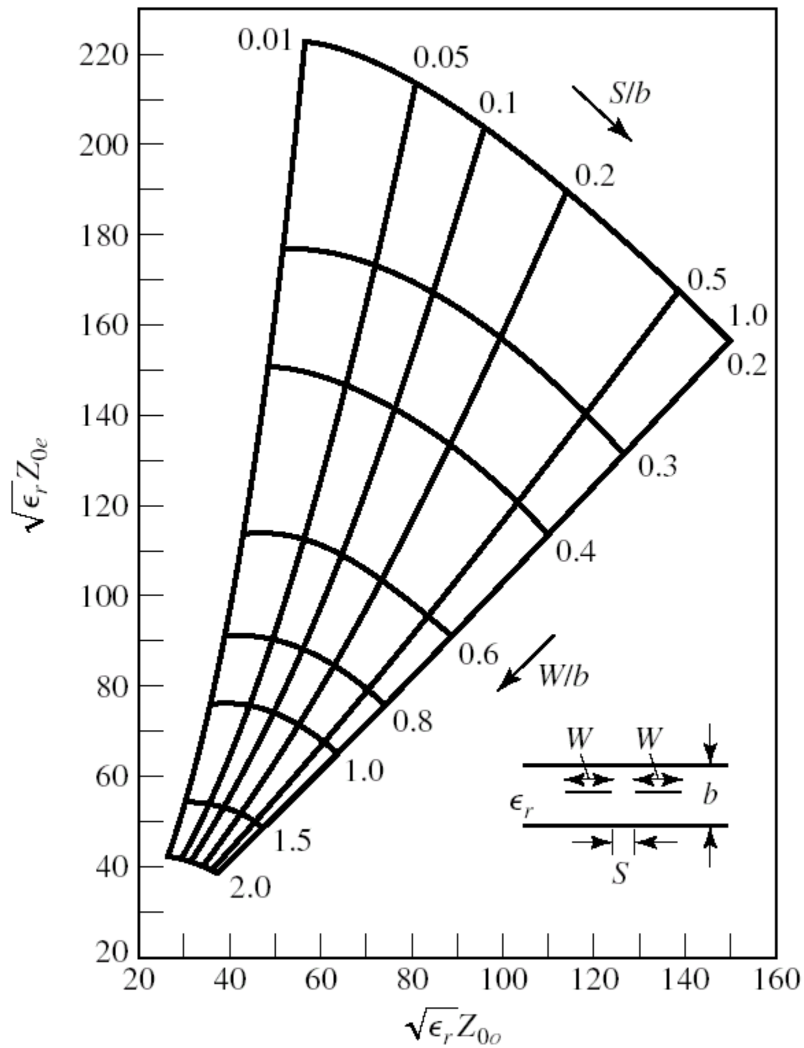


Figure 7.4

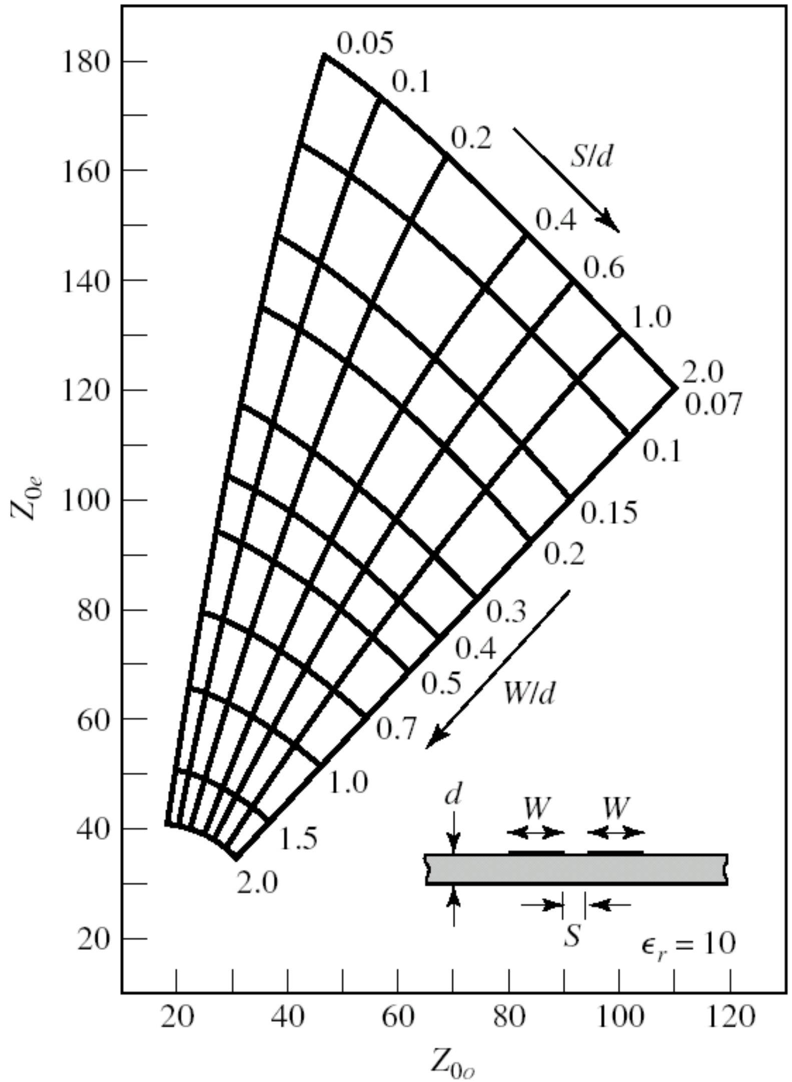
$$[S] = -j \cdot \begin{bmatrix} 0 & \sqrt{1-C^2} & jC & 0 \\ \sqrt{1-C^2} & 0 & 0 & jC \\ jC & 0 & 0 & \sqrt{1-C^2} \\ 0 & jC & \sqrt{1-C^2} & 0 \end{bmatrix}$$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

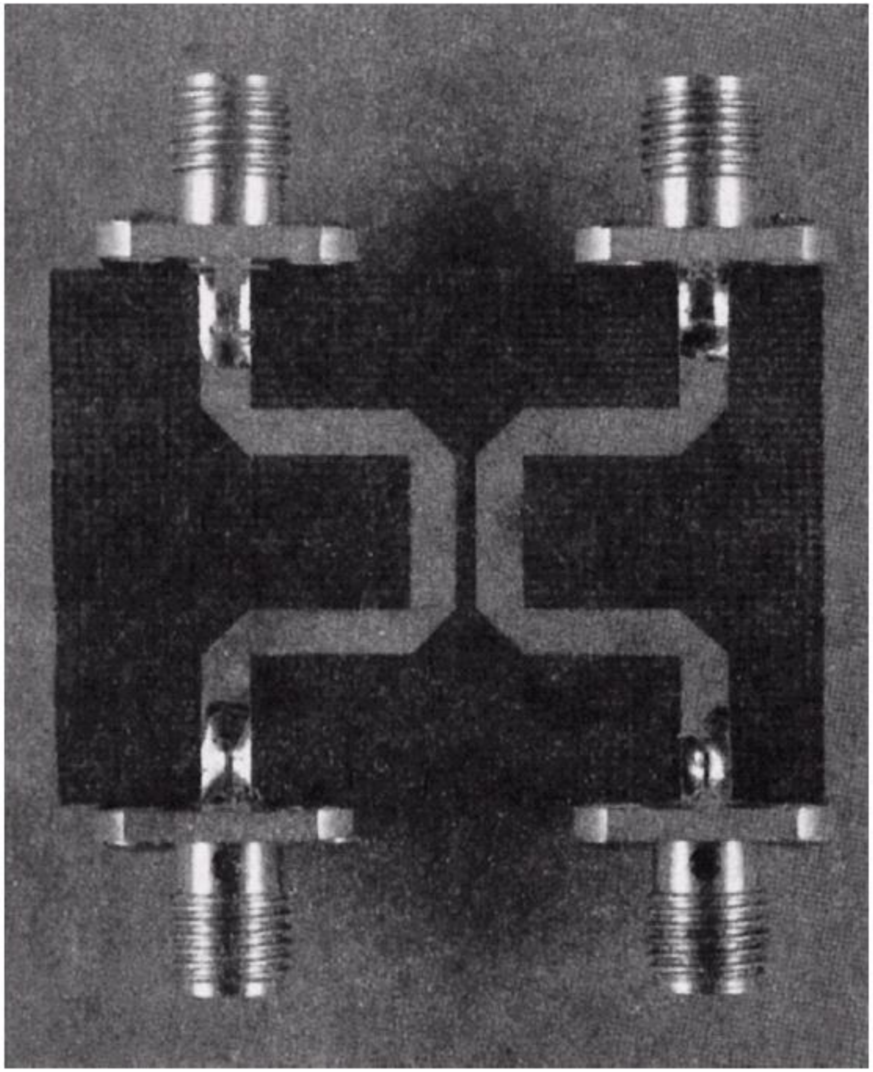
Normalized even- and odd-mode characteristic impedance design data for edge-coupled striplines.



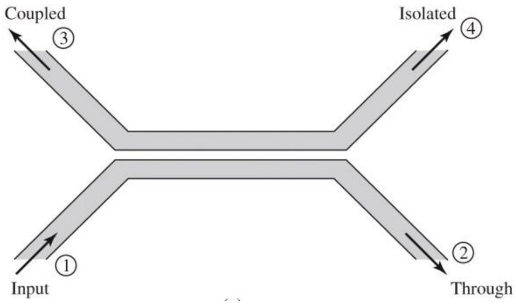
Even- and odd-mode characteristic impedance design data for coupled microstrip lines on a substrate with $\varepsilon_r = 10$.



Coupled Line Coupler



Coupled Line Coupler



$$\boxed{Z_{ce} Z_{co} = Z_0^2}$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$\boxed{C\,[\text{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)}$$

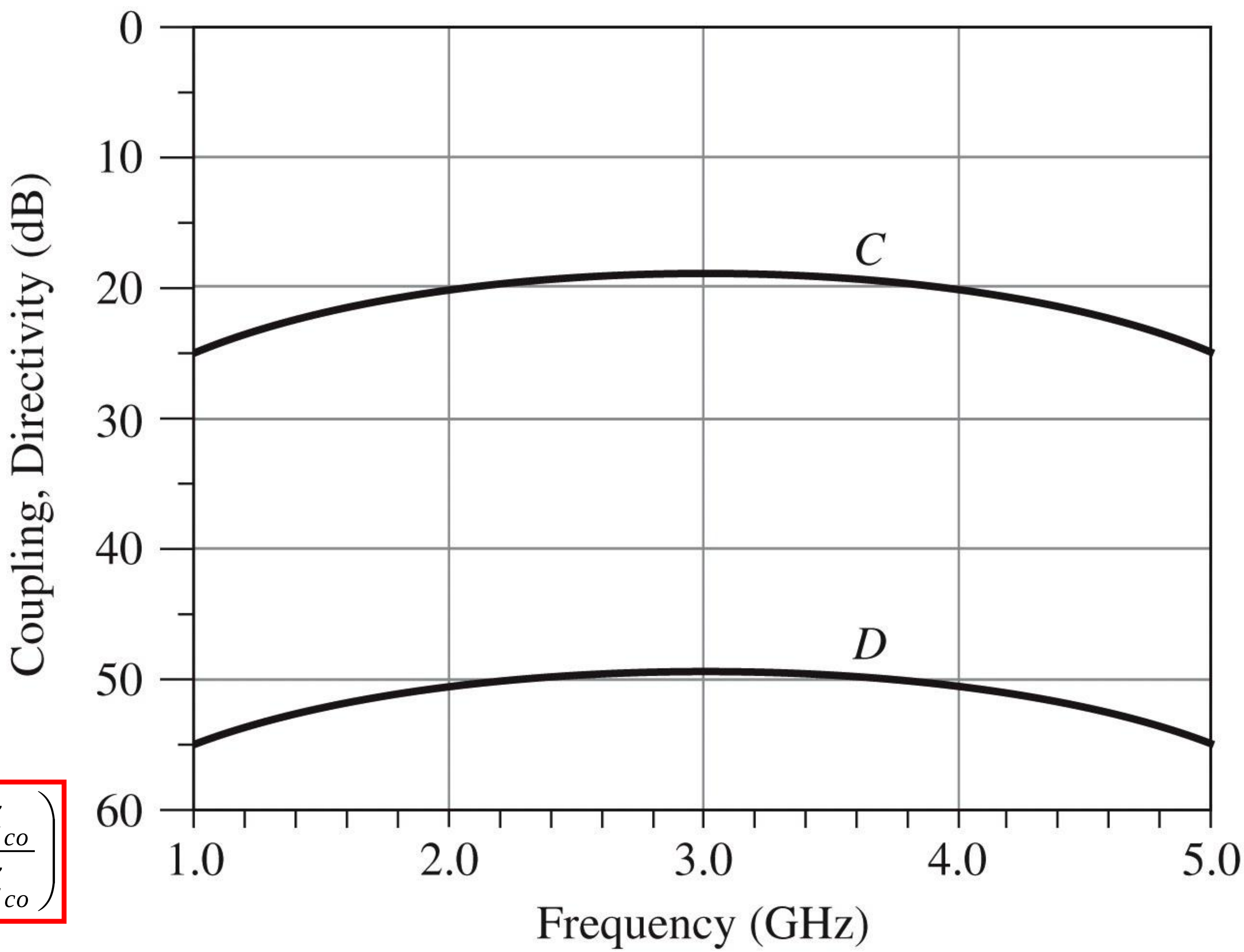


Figure 7.34

Example

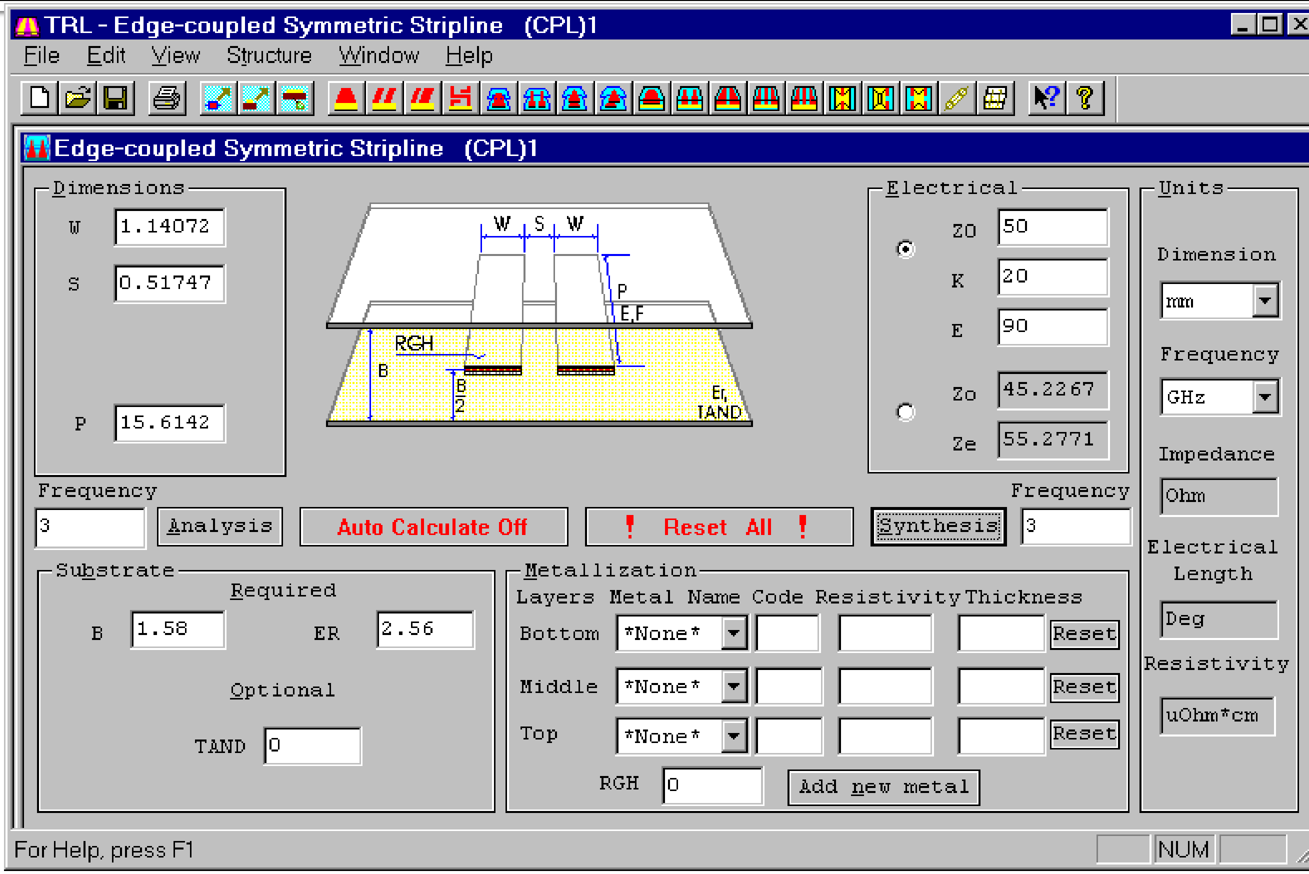
Design a coupled line coupler with 20 dB coupling factor, using stripline technology, with a distance between ground planes of 0.158 cm and an electrical permittivity of 2.56, working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz.

Solution

$$C = 10^{-20/20} = 0.1$$

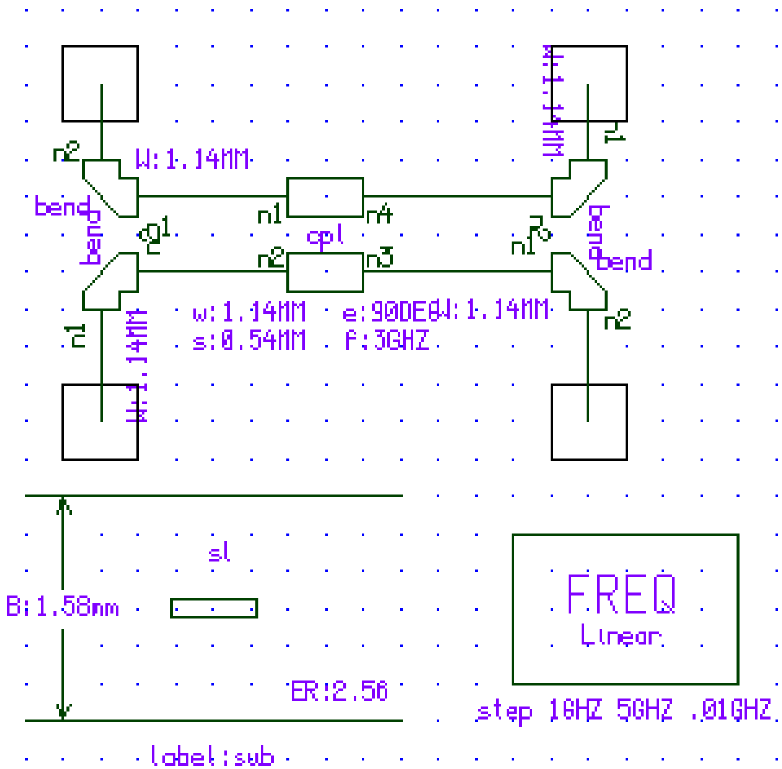
$$Z_{co} = 50\sqrt{\frac{0.9}{1.1}} = 45.23\Omega$$

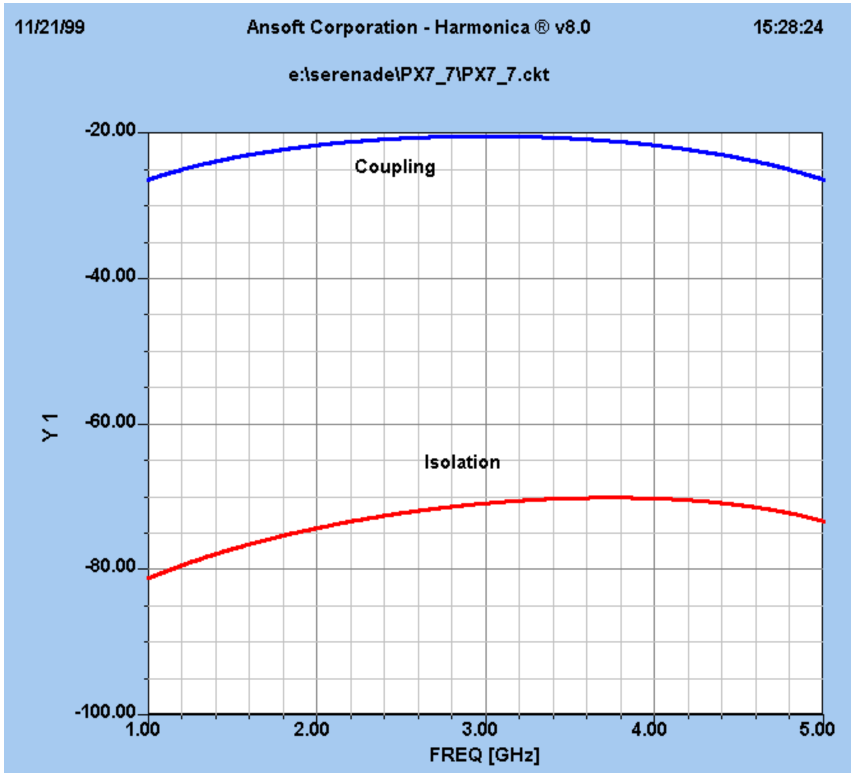
$$Z_{ce} = 50\sqrt{\frac{1.1}{0.9}} = 55.28\Omega$$



$$Z_{ce} = Z_0\sqrt{\frac{1+C}{1-C}}\,,\; Z_{co} = Z_0\sqrt{\frac{1-C}{1+C}}$$

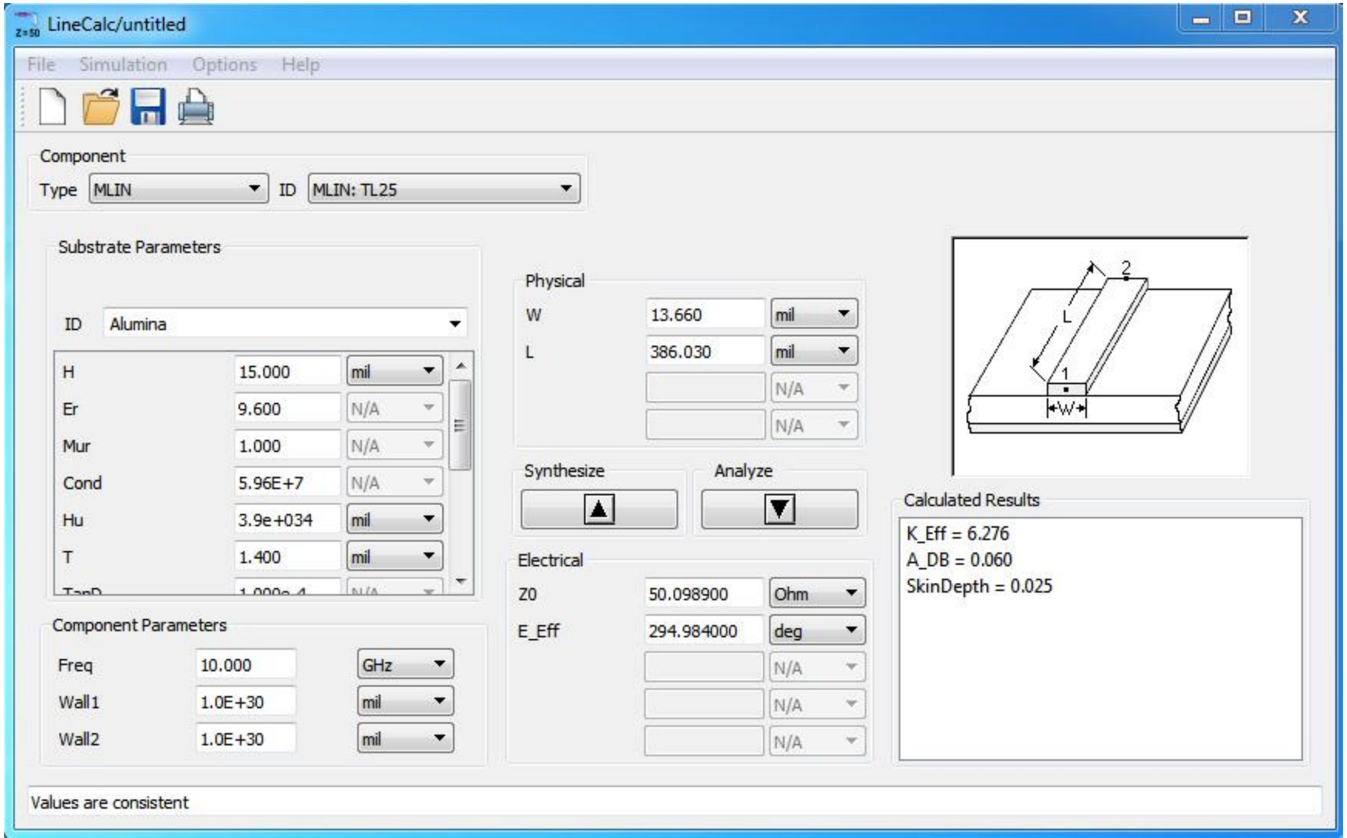
Simulation


W:1.14MM
bend
cpl
n1
n2
n3
n4
w:1.14MM
s:0.54MM
e:90DEG
W:1.14MM
f:3GHZ
B:1.58mm
sl
ER:2.56
label:sub
FREQ
Linear
step 1GHZ 5GHZ .01GHZ


11/21/99
Ansoft Corporation - Harmonica ® v8.0
15:28:24
e:\serenade\PX7_7\PX7_7.ckt
Coupling
Isolation
Y 1
-20.00
-40.00
-60.00
-80.00
-100.00
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

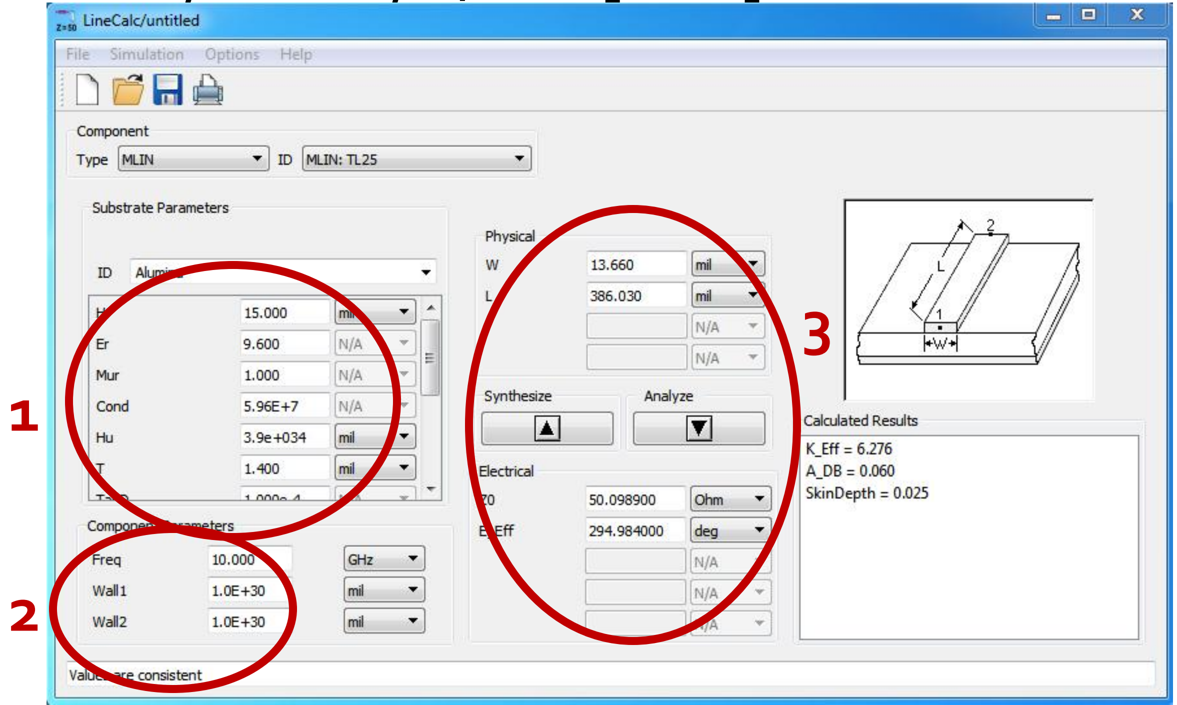
ADS linecalc

- In schematics: >Tools>LineCalc>Start
- for Microstrip lines >Tools>LineCalc>Send to Linecalc



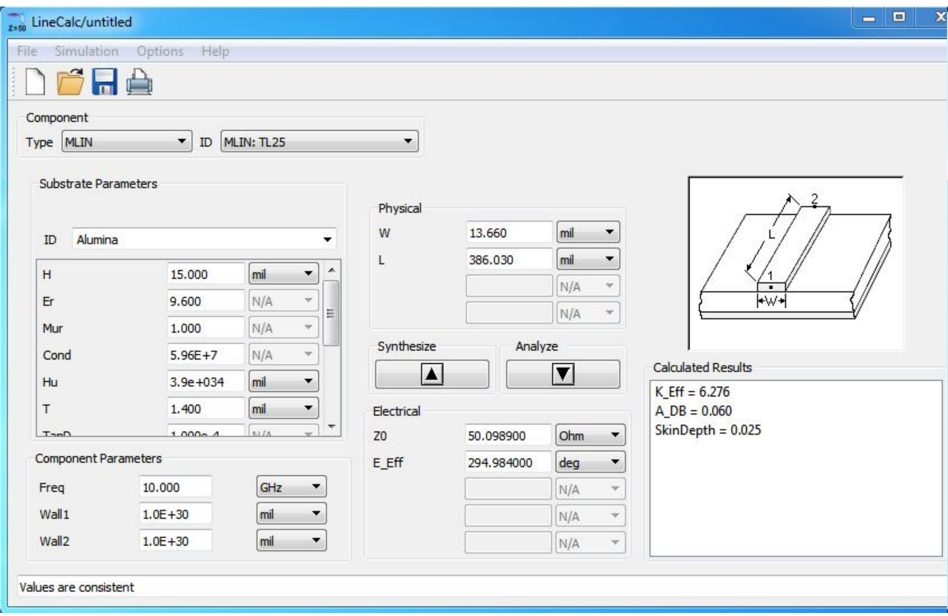
ADS linecalc

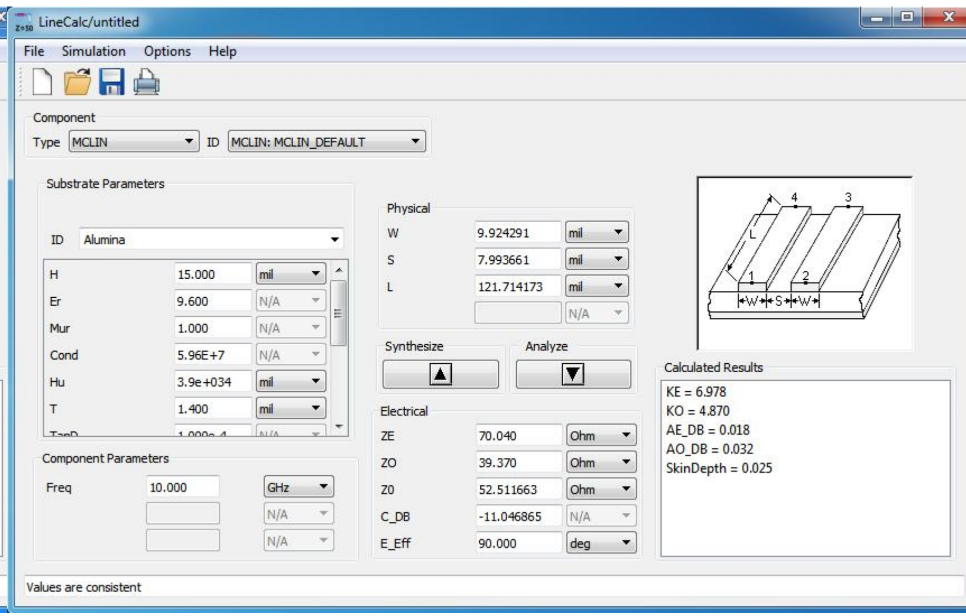
- 1. Define substrate (receive from schematic)
- 2. Insert frequency
- 3. Insert input data
  - Analyze: W,L → Zo,E or Ze,Zo,E / at f [GHz]
  - Synthesis: Zo,E → W,L / at f [GHz]



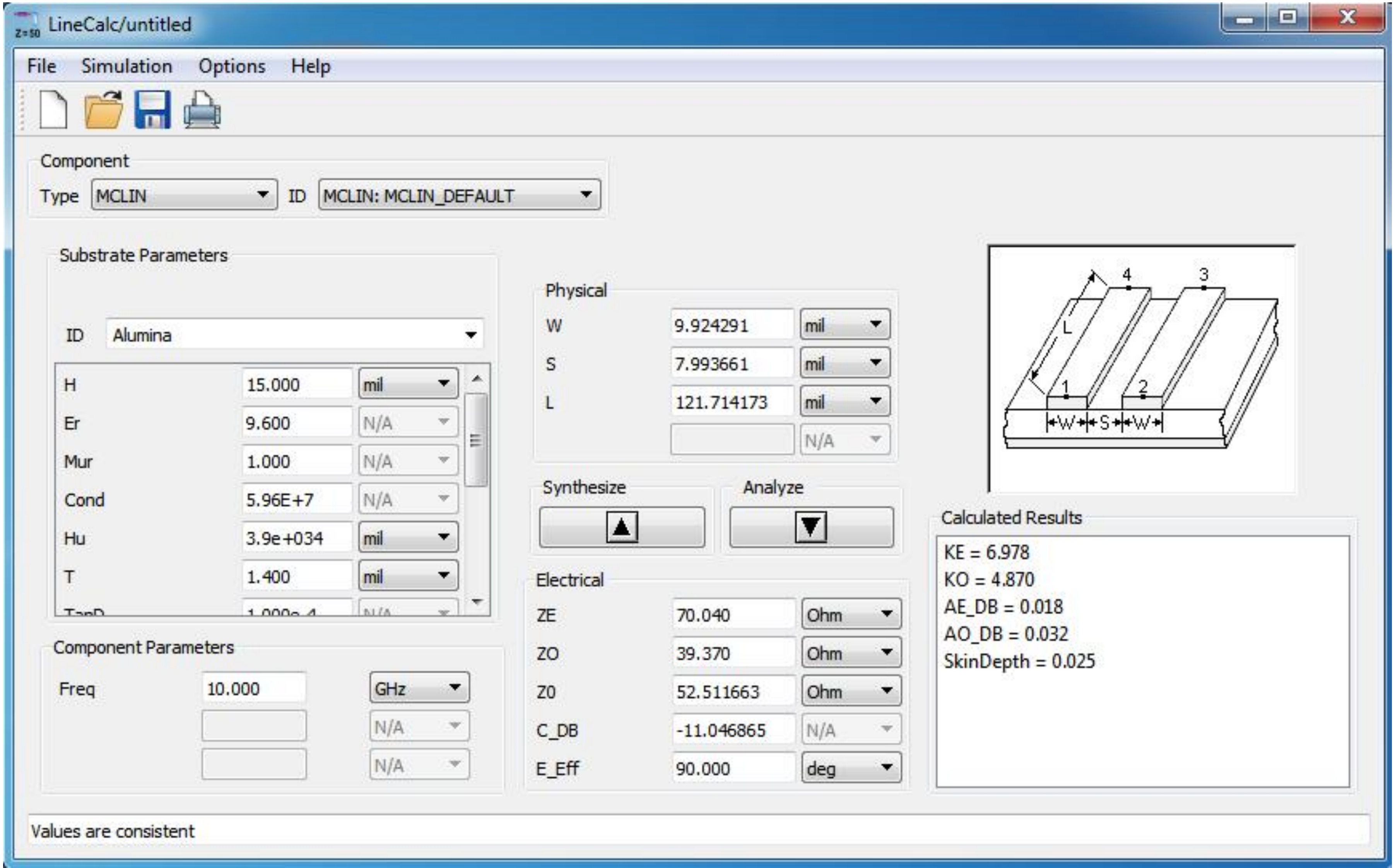
ADS linecalc

- Can be used for:
  - microstrip lines MLIN: W,L ⇔ Zo,E
  - microstrip coupled lines MCLIN: W,L,S ⇔ Ze,Zo,E





ADS linecalc



Multisection Coupled Line Couplers

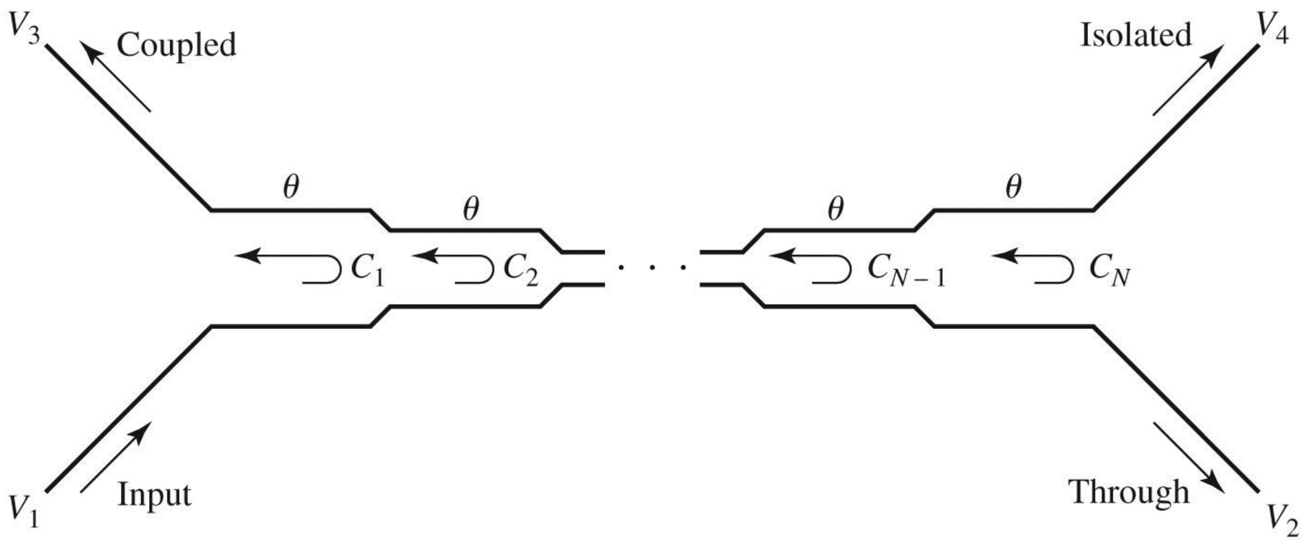


Figure 7.35

C<<1

$$\frac{V_3}{V_1} = b_3 = \frac{jC\sin\theta}{\cos\theta\sqrt{1-C^2} + j\sin\theta} = \frac{jCtg\theta}{\sqrt{1-C^2} + jtg\theta} \approx \frac{jCtg\theta}{1+jtg\theta} = jC\sin\theta e^{-j\theta}$$

$$\frac{V_2}{V_1} = b_2 = \frac{\sqrt{1-C^2}}{\cos\theta\sqrt{1-C^2} + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin} = e^{-j\theta}$$

$$C = \frac{V_3}{V_1} = 2j\sin\theta e^{-j\theta} e^{-j(N-1)\theta}\left[C_1\cos(N-1)\theta + C_2\cos(N-3)\theta + \ldots + \frac{1}{2}C_{\frac{N+1}{2}}\right]$$

Example

Design a three sections coupled line coupler with 20 dB coupling factor, binomial characteristic (maximum flat), working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz

Solution

$$\left.\frac{d^n}{d\theta^n}C(\theta)\right|_{\theta=\pi/2}=0,\ n=1,2$$

$$C=\left|\frac{V_3}{V_1}\right|=2\sin\theta\left[C_1\cos 2\theta+\frac{1}{2}C_2\right]=C_1(\sin 3\theta-\sin\theta)+C_2\sin\theta$$

$$\frac{dC}{d\theta}=\left[3C_1\cos 3\theta+(C_2-C_1)\cos\theta\right]\Big|_{\theta=\pi/2}=0$$

$$\frac{d^2C}{d\theta^2}=\left[-9C_1\sin 3\theta-(C_2-C_1)\sin\theta\right]\Big|_{\theta=\pi/2}=10C_1-C_2=0$$

$$\begin{cases} C_2-2C_1=0.1 \\ 10C_1-C_2=0 \end{cases}$$

$$\begin{cases} C_1=C_3=0.0125 \\ C_2=0.125 \end{cases}$$

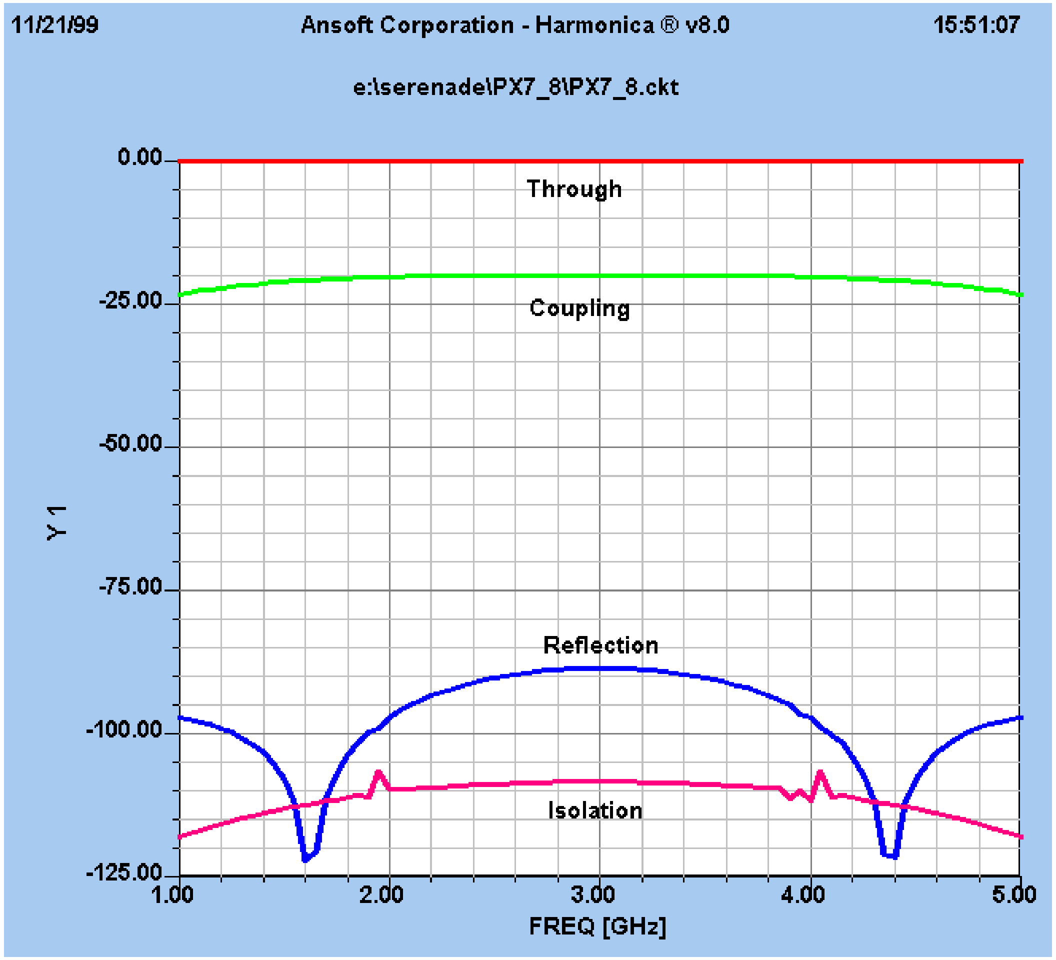
$$Z_{0e}^1=Z_{0e}^3=50\sqrt{\frac{1.0125}{0.9875}}=50.63\,\Omega$$

$$Z_{0o}^1=Z_{0o}^3=50\sqrt{\frac{0.9875}{1.0125}}=49.38\,\Omega$$

$$Z_{0e}^2=50\sqrt{\frac{1.125}{0.875}}=56.69\,\Omega$$

$$Z_{0o}^2=50\sqrt{\frac{0.875}{1.125}}=44.10\,\Omega$$

Simulare



11/21/99
Ansoft Corporation - Harmonica ® v8.0
15:51:07
e:\serenade\PX7_8\PX7_8.ckt
0.00
-25.00
-50.00
-75.00
-100.00
-125.00
Y 1
Through
Coupling
Reflection
Isolation
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

The Lange Coupler

- allows achieving coupling factors of 3 or 6 dB

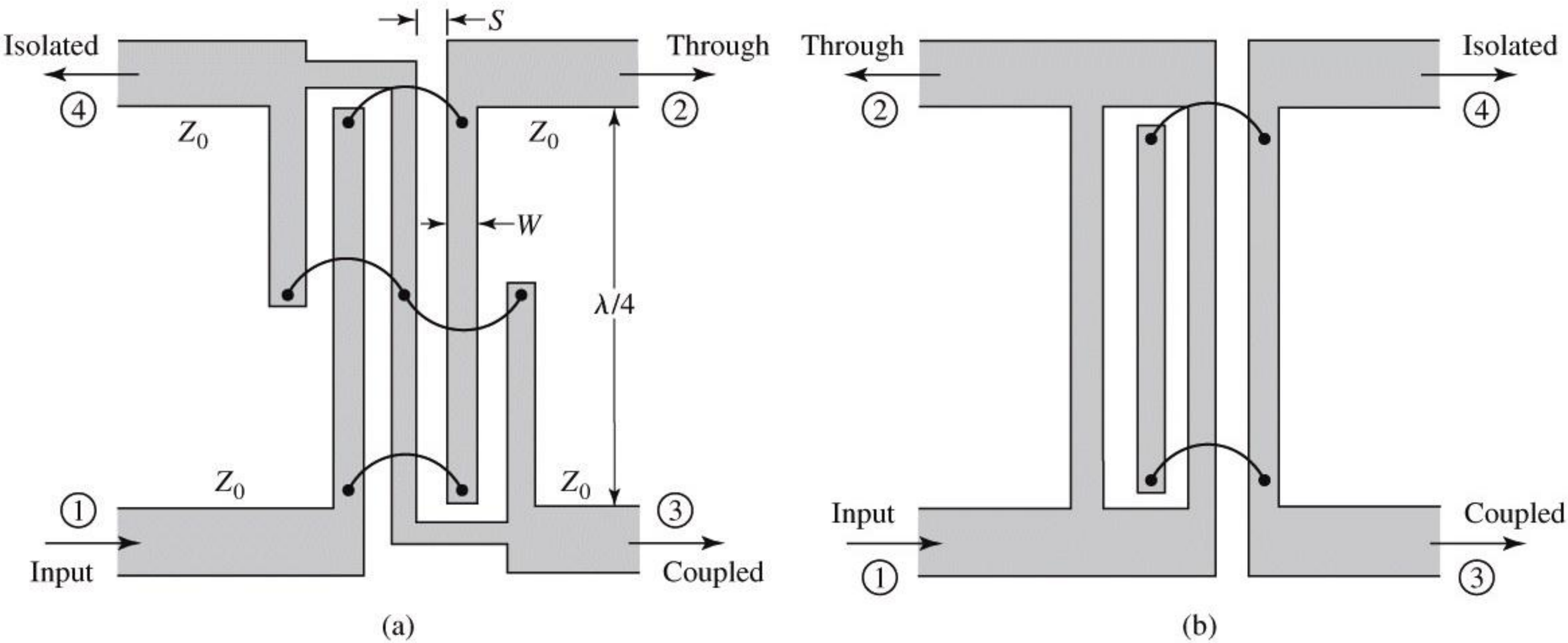


Figure 7.38

The Lange Coupler

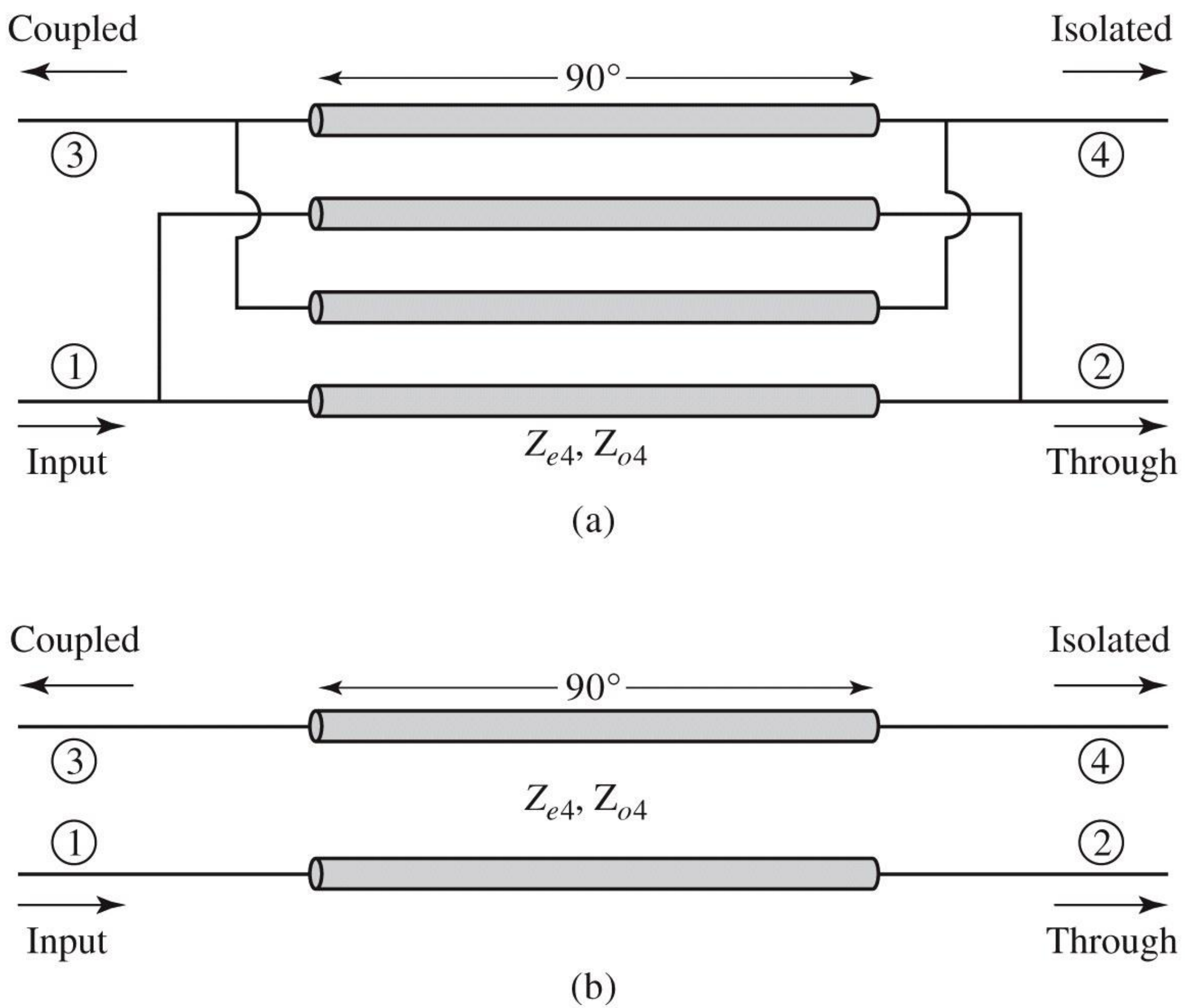
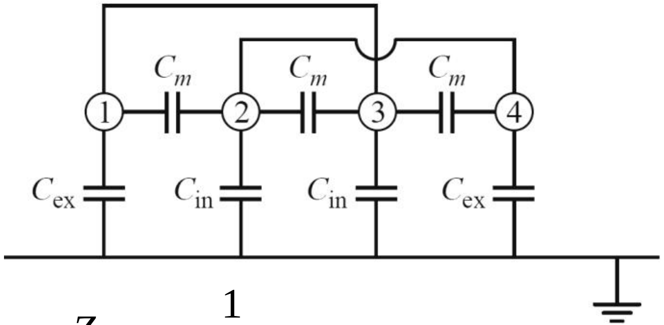
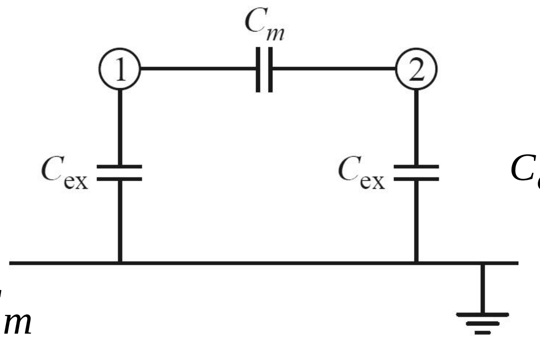


Figure 7.39

Circuit model



$$C_{in} = C_{ex} - \frac{C_{ex} C_m}{C_{ex} + C_m}$$



$$C_e = C_{ex}$$

$$C_o = C_{ex} + 2C_m$$

$$C_{e4} = C_{ex} + C_{in}$$

$$C_{o4} = C_{ex} + C_{in} + 6C_m$$

$$Z_{e4} = \frac{1}{vC_{e4}}$$

$$Z_{o4} = \frac{1}{vC_{o4}}$$

$$Z_0 = \sqrt{Z_{e4} Z_{o4}} = \sqrt{\frac{Z_{0e} Z_{0o} \left(Z_{0o} + Z_{0e}\right)^2}{\left(3Z_{0o} + Z_{0e}\right)\left(3Z_{0e} + Z_{0o}\right)}}$$

$$C_{e4} = \frac{C_e \left(3C_e + C_o\right)}{C_e + C_o}$$

$$Z_{e4} = Z_{0e} \frac{Z_{0e} + Z_{0o}}{3Z_{0o} + Z_{0e}}$$

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} = \frac{3\left(Z_{0e}^2 - Z_{0o}^2\right)}{3\left(Z_{0e}^2 + Z_{0o}^2\right) + 2Z_{0e} Z_{0o}}$$

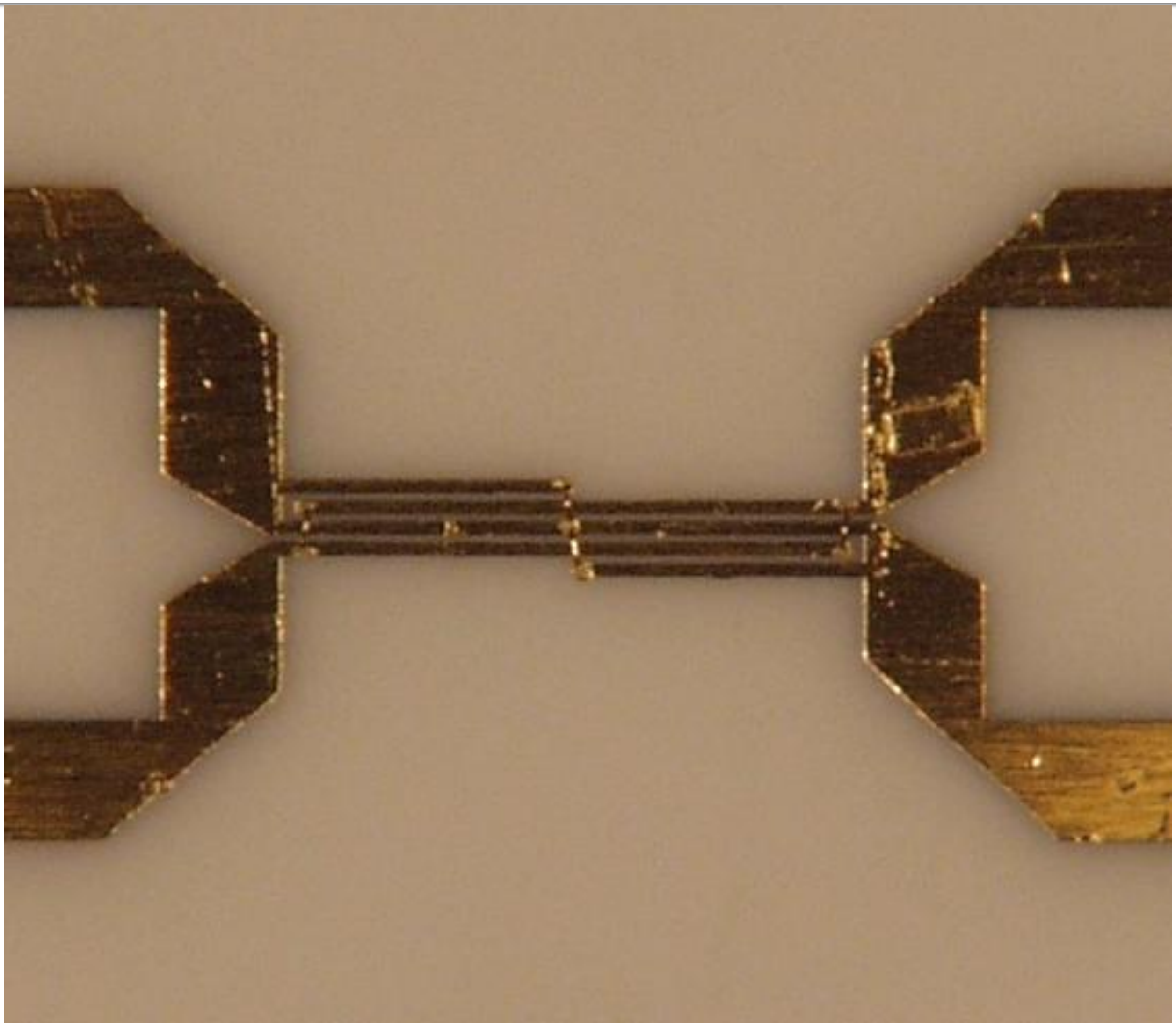
$$C_{o4} = \frac{C_o \left(3C_o + C_e\right)}{C_e + C_o}$$

$$Z_{o4} = Z_{0o} \frac{Z_{0e} + Z_{0o}}{3Z_{0e} + Z_{0o}}$$

$$\boxed{Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{\left(1 - C\right)/\left(1 + C\right)}} Z_0}$$

$$\boxed{Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{\left(1 + C\right)/\left(1 - C\right)}} Z_0}$$

The Lange Coupler



Directional Couplers

Laboratory no. 2

Directional Coupler

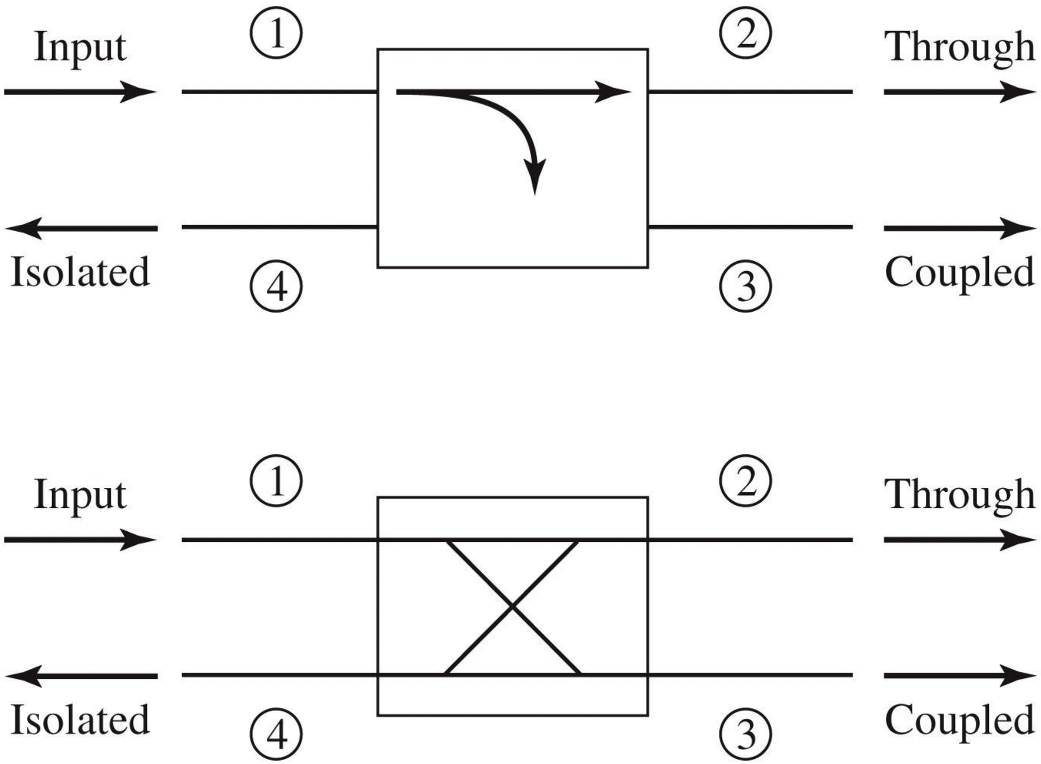


Figure 7.4

$$\left|S_{12}\right|^2 = \alpha^2 = 1 - \beta^2$$

$$\left|S_{13}\right|^2 = \beta^2$$

Cuplaj

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivitate

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{\left|S_{14}\right|}\right)[\mathrm{dB}]$$

Izolare

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log\left|S_{14}\right|\,[\mathrm{dB}]$$

$$I = D + C\;,\;\; dB$$

The cuadrature (90°) hybrid

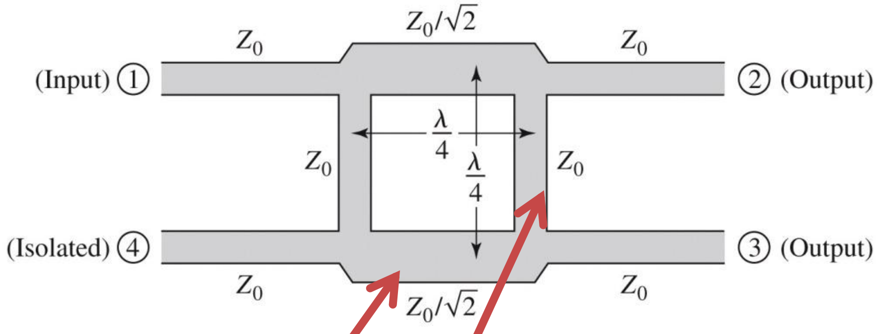


Figure 7.21

$$\boxed{y_2^2 = 1 + y_1^2}$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$\boxed{C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}}$$

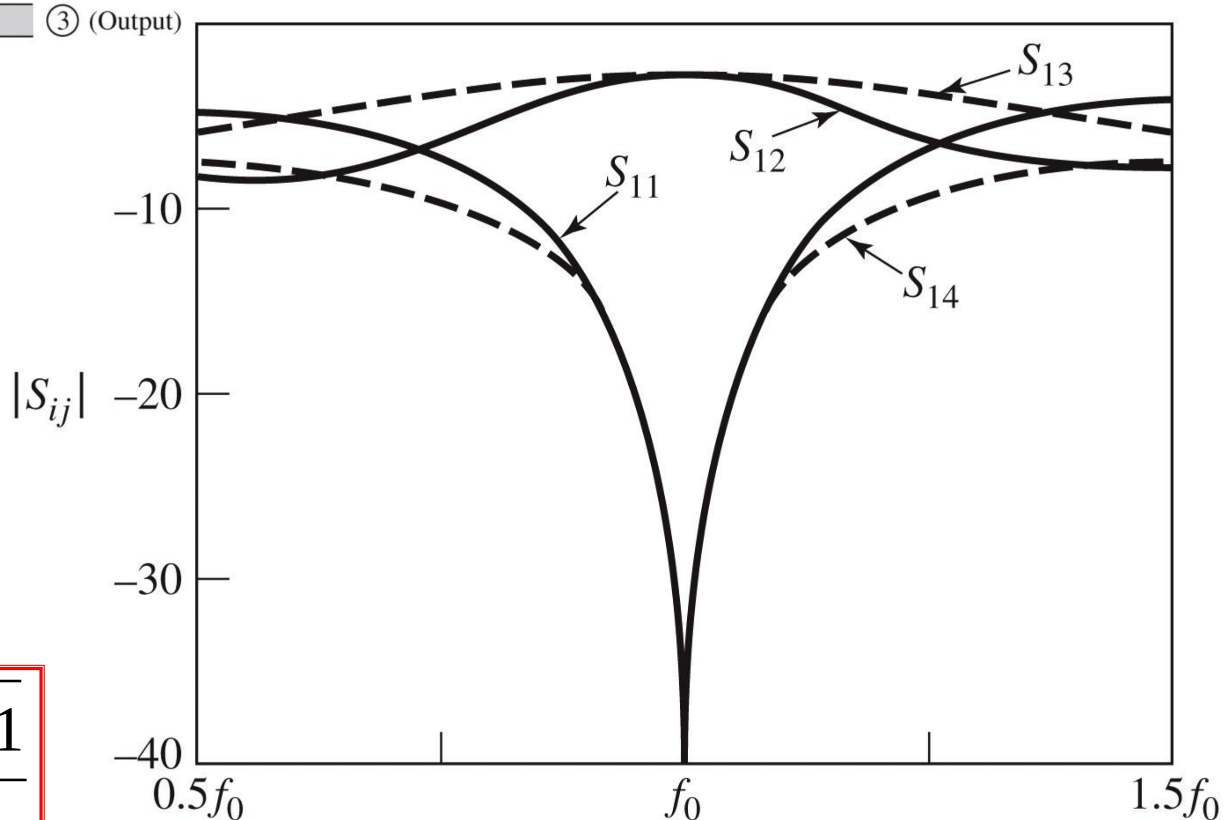
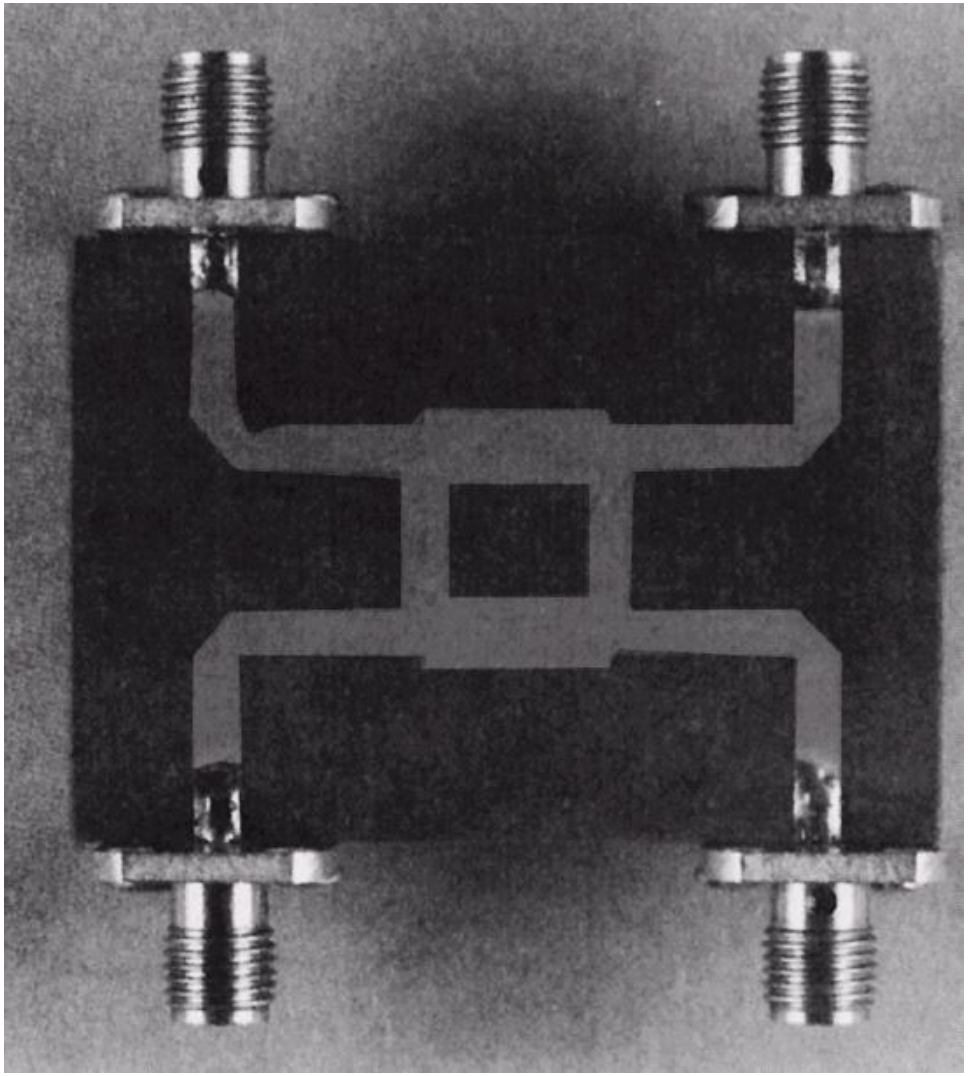
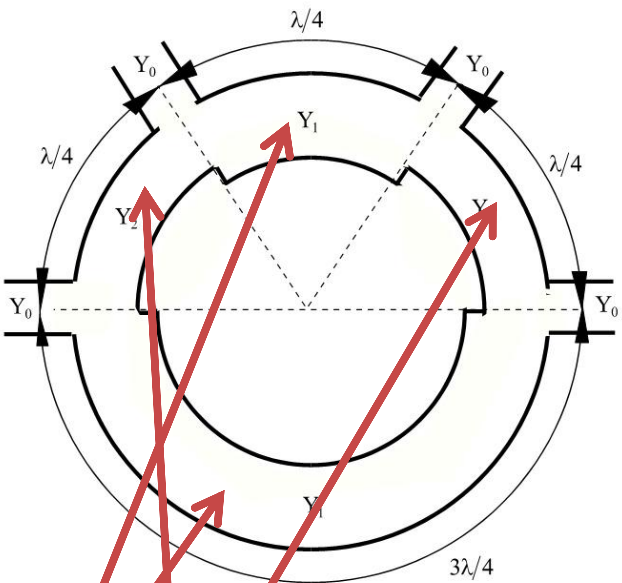


Figure 7.25

Quadrature coupler



The 180° ring hybrid (rat-race)



$$\boxed{y_1^2 + y_2^2 = 1}$$

$$\left|\beta\right| = y_1$$

$$\boxed{C\,[\mathrm{dB}] = -20 \cdot \log_{10}(y_1)}$$

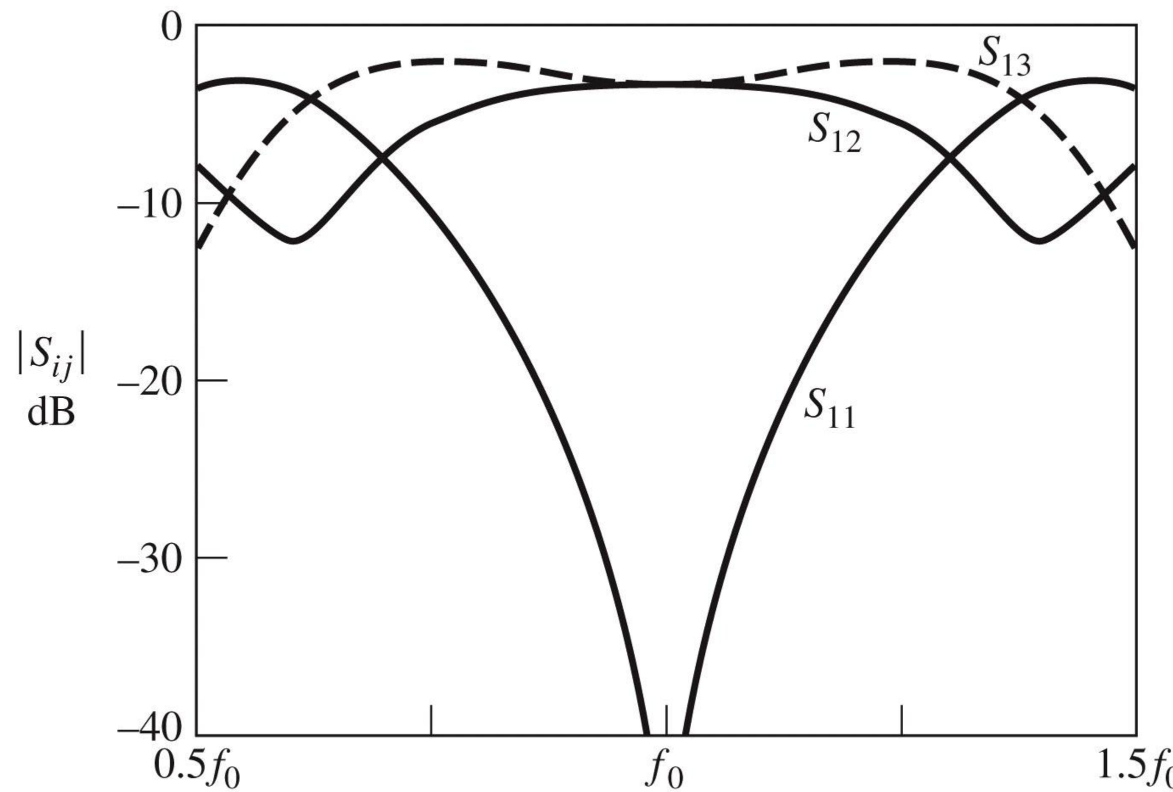


Figure 7.46

Ring coupler

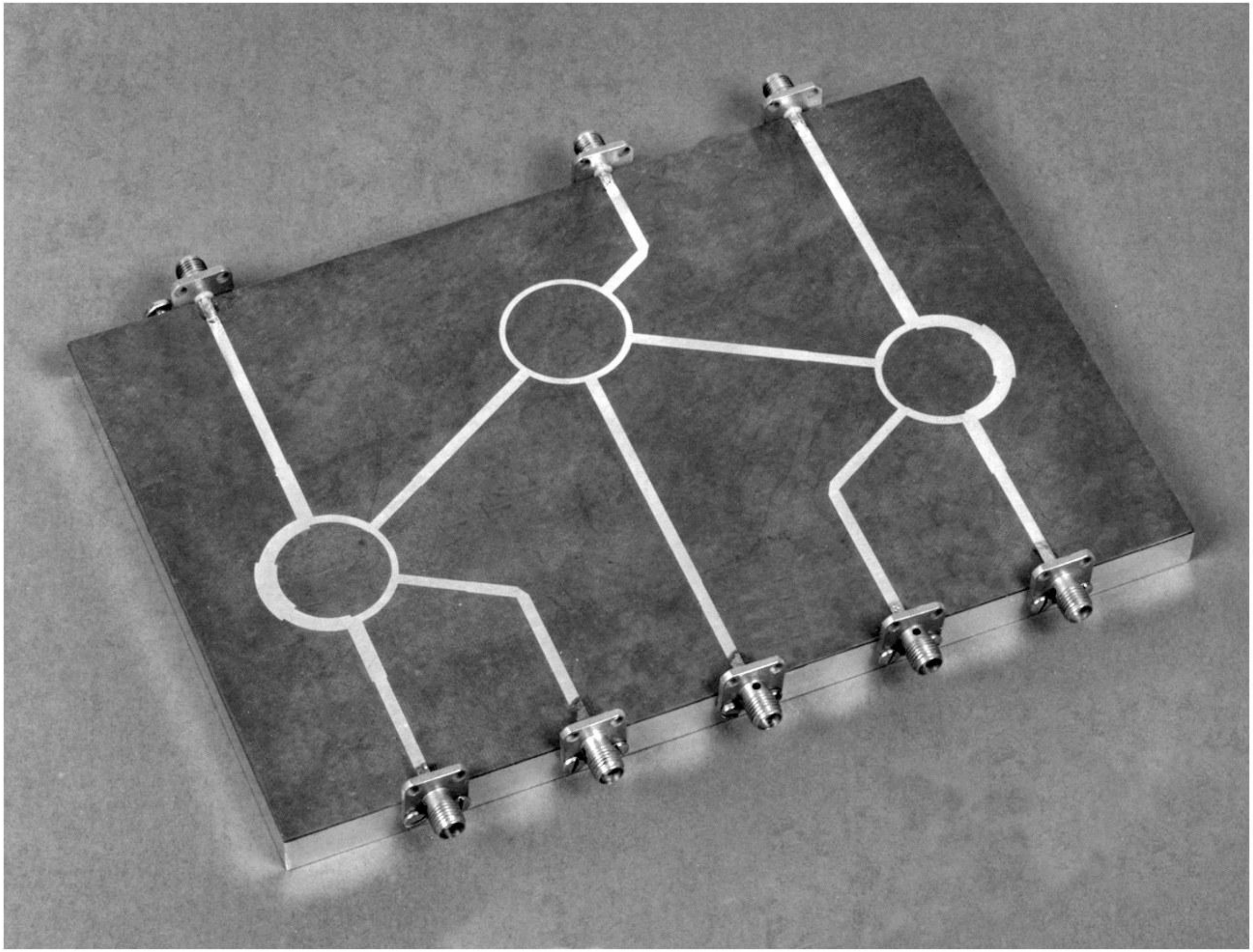
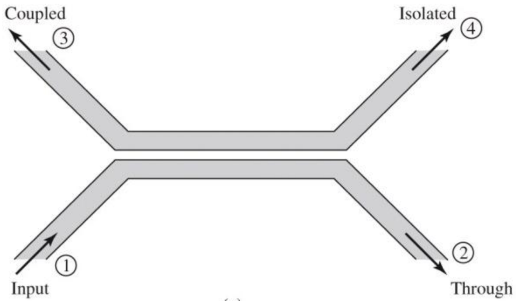


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler



$$\boxed{Z_{ce} Z_{co} = Z_0^2}$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$\boxed{C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)}$$

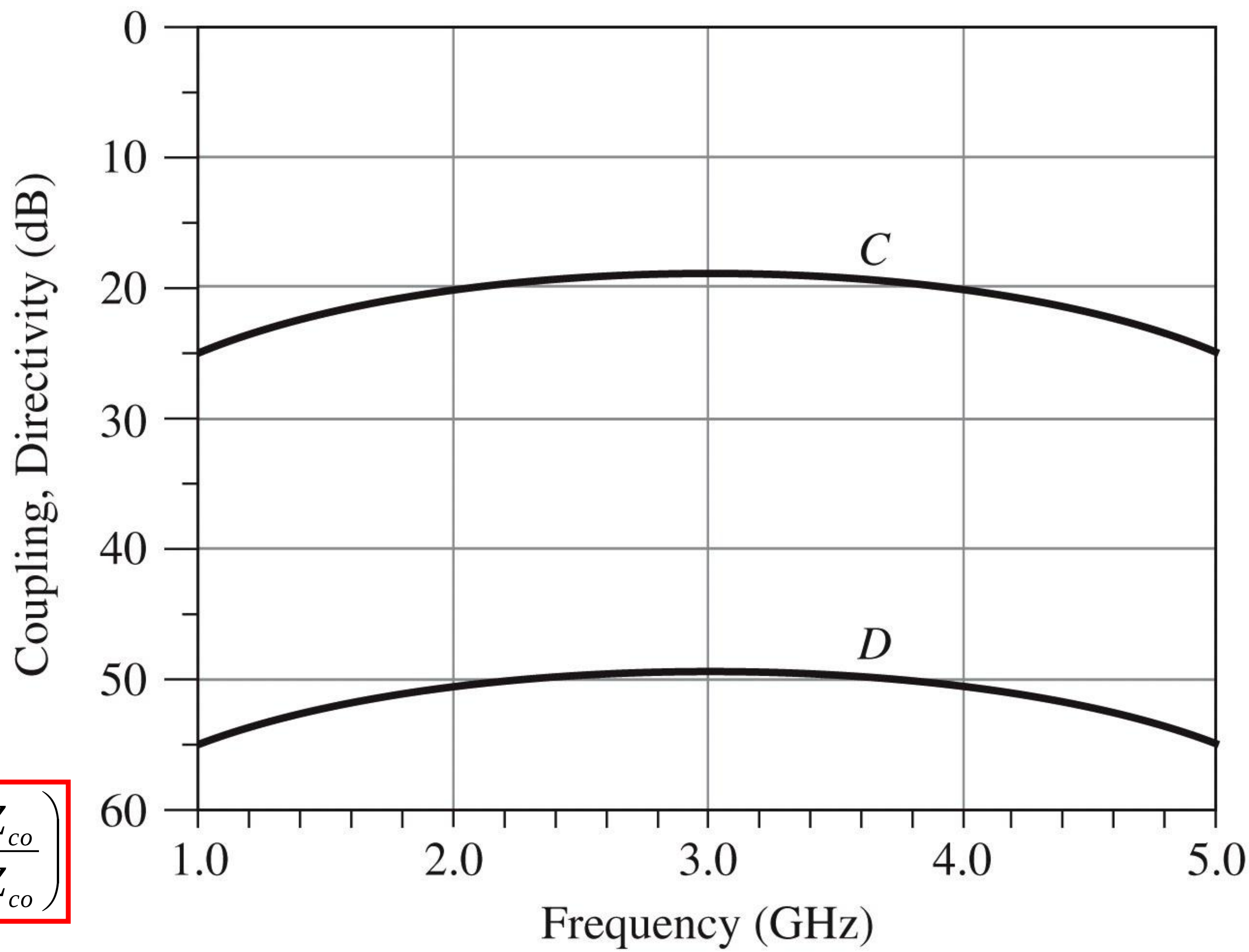
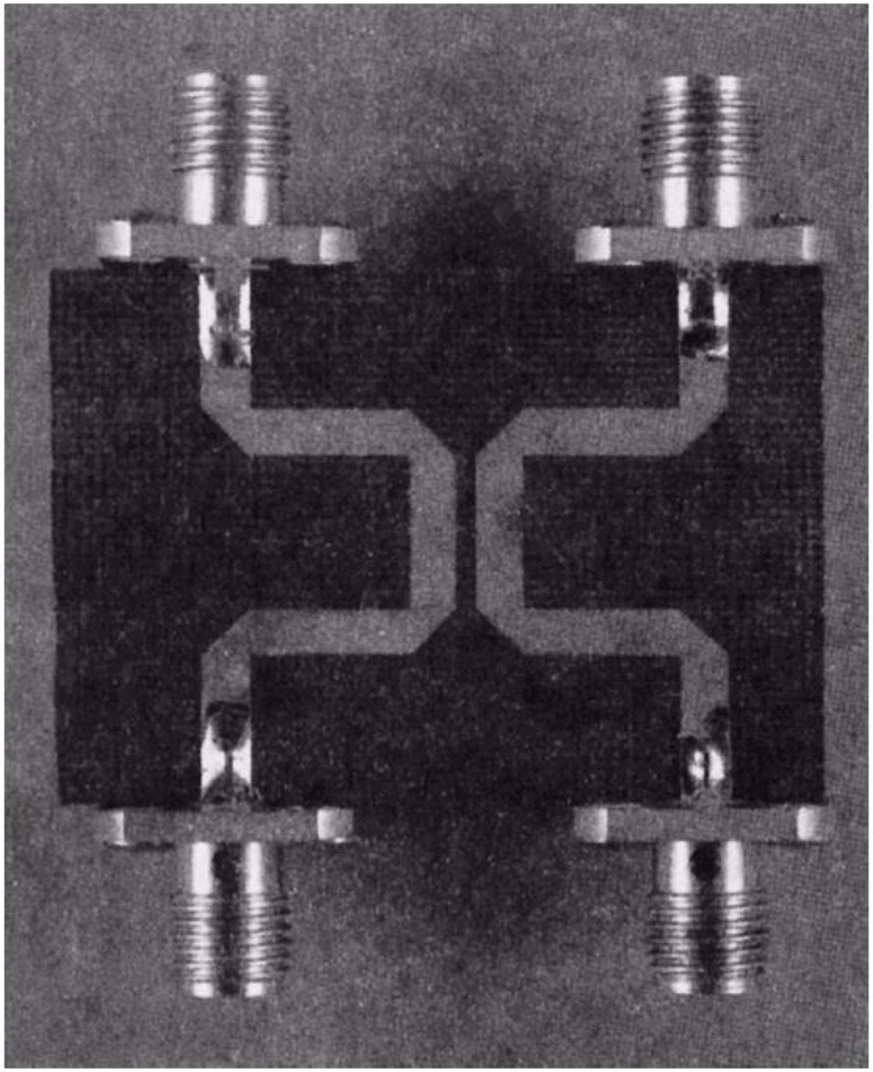


Figure 7.34

Coupled line coupler



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